Book of Abstracts

3rd International Conference on Pure and Applied Mathematics

ICPAM – VAN 2020

September 3-5, 2020

Van - TURKEY

Van Yüzüncü Yıl University

3rd International Conference on Pure and Applied Mathematics (ICPAM-VAN 2020)

BOOK OF ABSTRACTS

Van, TURKEY
September 3-5, 2020

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Welcome

Dear Participants,

Welcome to 3rd International Conference on Pure and Applied Mathematics, ICPAM - VAN 2020, Van, Turkey. The conference is organized at Van Yüzüncü Yıl University from September 3rd to September 5th. Since safety and health of our guests, staff and community are of utmost importance to us, at this time, we have made the difficult decision to postpone all travels, as well as participations in ICPAM - VAN 2020 that involve large gatherings. Due to the Covid-19 concerns, this year the third ICPAM - VAN 2020 becomes a virtual conference, that is, it turns into an interactive seminar conducted over the internet (webinar) while the first and the second ones were organized as face to face meetings in 2015 and 2018, respectively. The reason for our 3-year long break is to be a host of another high involvement conference.

The purpose of the virtual conference is to provide a platform where researchers in the field of pure and applied mathematics can present their researches, exchange new ideas, discuss challenging issues, foster future collaborations and interact with each other during these difficult days.

With 9 invited speaker from 7 different countries, 5 parallel sessions (totally 20 sessions) and 97 presentations and more than 100 participants from 13 countries, Austria, Czechia, Germany, India, Iraq, Iran, Italy, Jordan, Kuwait, Nigeria, Pakistan, Russia, Turkey, United Arab Emirates, as well as people from 35 different university from Turkey, ICPAM - VAN 2020 will provide a stimulating opportunity for a global interchange of ideas on recent advances in mathematics.

I would like to express my deep gratitude to Prof. Dr. Hamdullah ŞEVLİ, President of Van Yüzüncü Yıl University, for his encouragement and support in all stages of this conference.

I am grateful to all the participants in the International Conference on Pure and Applied Mathematics, particularly the members of the Scientific and Organizing Committees, the referees and the authors for producing such a high standard conference.

I would also like to thank to the sponsors, Van Yüzüncü Yıl University and Abdullah Gül University for their generous support.

The last but not the least, I would like to give our heartful thanks to participants for their understanding, patience and contributions which make our conference more successful. Since the conference is almost entirely from the registration support of participants, I am grateful for their financial support as well. Thank you all again for the unbelievable amount of support and understanding you have shared during this period. It means more than words can express. We, the members of organizing committee and I, were honored and happy to organize this online meeting which provides online interactions between mathematicians who can not meet, talk and discuss face to face and offers an online collaboration and online social networks.

I wish all of you to stay healthy and safe. I look forward to the day we welcome you all back to Van, city of the sun.

Professor Cemil Tunç
Chair Organizer of ICPAM - VAN Organizing Committee

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10:15-11:00	ALIREZA KHALILI GOLMANKHANEH	Invited Speaker			
11:00-11:45	ANDRÁS RONTÓ	Invited Speaker			
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Chair	**CEMİL TUNÇ**				
13:30-14:15	HANS-PETER SCHRÖCKER	Invited Speaker			
14:15-15:00	AYMAN BADAWI	Invited Speaker			
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Abstracts of invited speakers

Absorbing and weakly absorbing ideals of commutative rings

AYMAN BADAWI

The American University of Sharjah, Sharjah, United Arab Emirates

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Let R be a commutative ring with $1 \neq 0$. Recall that a proper ideal I of R is called a *2-absorbing ideal* of R if $a, b, c \in R$ and $abc \in I$, then $ab \in I$ or $ac \in I$ or $bc \in I$. A more general concept than 2-absorbing ideals is the concept of n-absorbing ideals. Let $n \geq 1$ be a positive integer. A proper ideal I of R is called an *n-absorbing ideal* of R if $a_1, a_2, ..., a_{n+1} \in R$ and $a_1 a_2 \cdots a_{n+1} \in I$, then there are n of the a_i's whose product is in I. The concept of n-absorbing ideals is a generalization of the concept of prime ideals (note that a prime ideal of R is a 1-absorbing ideal of R). In this talk, we will state recent developments on the study of absorbing ideals of commutative rings.

MSC 2010: 13A05, 13F05, 05A15, 13G05
Keywords: prime ideal, primary ideal, 1-absorbing primary ideal, 2-absorbing primary ideal, 2-absorbing ideal, weakly prime ideal, weakly primary ideal, weakly 2-absorbing primary ideal, weakly semiprime ideal, n-absorbing ideal

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The framework of input-to-state stability with application to interconnected systems

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The notion of input-to-state stability (ISS) was introduced in [1] for nonlinear finite dimensional systems of the form

$$\dot{x} = f(x, d), \quad x(t) \in \mathbb{R}^n, \quad d(t) \in \mathbb{R}^m, \quad t \geq 0, \tag{1}$$

with an unknown disturbance $d \in L_\infty^{loc}$. This notion extend the usual Lyapunov stability to the case of systems with input signals. Several weaker versions of this notion appeared soon after that in the literature together with different characterizations of these notions. Later these notions were extended to other classes of systems, including discrete time systems, switched, impulsive and hybrid systems, systems with time delay as well as systems given in form of partial differential equations, see [2], [3].

This presentation we begin with an introduction to ISS and related notions illustrated by means of simple practical examples. Characterizations of these notions, in particular by means of Lyapunov functions, will be provided and illustrated as well. We will show that this framework is very fruitful for stability investigation of nonlinear networks. We will explain how the small-gain theory in the ISS framework can be applied to study stability of interconnected systems, including large scale multi-agent systems.

MSC 2010: 37C75, 93D30, 93C10, 93C25
Keywords: Stability, nonlinear systems, interconnected systems, Input-to-state stability, multi-agent systems, Lyapunov methods

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On the existence and stability of symmetric solutions in a class of weakly non-linear systems

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For a class of systems of weakly non-linear ordinary differential equations

$$u'(t) = \varepsilon h(t, u(t)), \qquad t \in J,$$

where $J \subset \mathbb{R}$ is an open interval and ε is a small parameter, we study solutions satisfying the condition

$$u(\psi(t)) = Au(t), \qquad t \in J,$$

which, with different choices of the function $\psi : J \to J$ and the matrix A, includes various frequently encountered properties (e.g., oddness, periodicity, antiperiodicity or affine periodicity). It is assumed that the continuous function h is smooth in the space variable and satisfies a suitable invariance condition.

For solutions having the property mentioned, effective existence and asymptotic stability conditions are obtained, which are closely related to classical results. The asymptotic stability conditions are formulated in terms of either logarithmic norms or spectral abscissas of certain matrices that can be found explicitly.

We review the main results, outline the ideas of proof, and mention the relation to known statements.

MSC 2010: 34C14, 34C15, 34C25
Keywords: Odd solution, periodic solution, anti-periodic solution, averaging, stability

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On Hardy, Copson, Bennett, Leindler type dynamic inequalities

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Hardy's celebrated inequalities [1, 2] in both discrete and continuous forms gave inspirations to many mathematicians such as Copson, Bennett, Leindler and so on. They generalized and improved Hardy's inequalities and their reverse versions, which are called Bennett-Leindler inequalities, for both cases seperately. In order to avoid proving results twice, once for continuous functions and once for functions defined on discrete sets, time scale unification of these inequalities have appeared in the literature by using delta and nabla derivatives and integrals. Since these unifications are not adequate in the theoretical research of some differential and difference equations and in certain computational applications such as adaptive computing and multiscale methods [3], the diamond alpha, $\diamond_\alpha$, calculus, which utilizes convex linear combinations of delta and nabla derivatives and integrals, has been introduced by Sheng et al. [3].

In this talk we give a survey for Hardy, Copson, Bennett and Leindler type dynamic inequalities on time scale delta and nabla calculii. Then we present two kinds of dynamic Bennett-Leindler type inequalities via the diamond alpha calculus. The former, which is composed of inequalities of mixed type containing delta, nabla and diamond alpha integrals together, strengthens and binds the existing results obtained for time scale delta and nabla calculii. The latter, which consists of inequalities including only diamond alpha integrals, harmonizes and unifies the foregoing results. Moreover these inequalities provide novel and better results in the special cases.

MSC 2010: 26D10, 26D15, 26E70
Keywords: Nabla derivative, Hardy inequality, Copson inequality, Bennett inequality, Leindler inequality

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Fractal calculus an extension of ordinary calculus

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Fractal calculus was formulated recently as an extension of standard calculus. This framework was generalized and applied in several branches of science and engineering. Fractal calculus, like ordinary calculus, has both local and non-local versions. An advantage of this type of fractal calculus is that it is algorithmic, which makes it more suitable for applications like ordinary calculus. Many frameworks for calculus on fractals were formulated, but some of them are ad-hoc or non-algorithmic and, as a result, difficult to utilize in science. Fractal calculus involves fractional derivatives that have geometrical as well as physical meaning. In this work, we present the local and nonlocal fractal calculus and applications in physics such as statistical mechanics and quantum physics.

MSC 2010: 28A80, 44A10, 58C35
Keywords: Local fractal derivatives, non-local fractal derivative, fractal calculus, Cantor-like set

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On hybrid problems with hybrid boundary value problems

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By mixing and using some idea, we are going to investigate the fractional hybrid problem for the thermostat model as

$$ {}^{c}\mathcal{D}_0^{p}\Big(\frac{z(t)}{h(t,z(t))}\Big) + \Phi(t,z(t)) = 0, \quad (t \in [0,1]) \tag{1} $$

with the hybrid boundary value conditions

$$ \begin{cases} \mathcal{D}\Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=0} = 0, \\ \lambda\, {}^{c}\mathcal{D}_0^{p-1}\Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=1} + \Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=\eta} = 0, \end{cases} \tag{2} $$

where $p \in (1,2]$, $p-1 \in (0,1]$, $\eta \in [0,1]$, $\mathcal{D} = \frac{\mathrm{d}}{\mathrm{d}t}$, λ is a positive real parameter, ${}^{c}\mathcal{D}_0^{\gamma}$ is the Caputo derivation of fractional order $\gamma \in \{p, p-1\}$, the function $\Phi : [0,1] \times \mathbb{R} \to \mathbb{R}$ is continuous and $h \in C([0,1] \times \mathbb{R}, \mathbb{R} \setminus \{0\})$.

Also, we investigate the hybrid fractional differential inclusion model of thermostat as

$$ -{}^{c}\mathcal{D}_0^{p}\Big(\frac{z(t)}{h(t,z(t))}\Big) \in \mathcal{G}(t,z(t)), \qquad t \in [0,1] \tag{3} $$

with three-point hybrid boundary value conditions

$$ \begin{cases} \mathcal{D}\Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=0} = 0 \\ \lambda\, {}^{c}\mathcal{D}_0^{p-1}\Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=1} + \Big(\frac{z(t)}{h(t,z(t))}\Big)\Big|_{t=\eta} = 0 \end{cases} \tag{4} $$

where, $\mathcal{G} : [0,1] \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multi-valued map via some properties.

MSC 2010: 26A33, 34A08, 34A12
Keywords: Caputo fractional derivative, hybrid fractional differential equation and inclusion, thermostat modeling

Rational kinematics

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A rigid body motion is called "rational" if all trajectories are rational curves [4]. Rational motions have been investigated since the second half of the 19th century but, nonetheless, some of their rather fundamental properties have been unveiled only in the past decade. In our talk we present an overview of these results, accessible for a general mathematical audience.

A central aspect is the parametrization of rational motion via certain polynomials over rings ("motion polynomials"). Factorization of motion polynomials [1, 3] corresponds to the decomposition of rational motions into sequences of rotations and has important applications in mechanism science. The trajectories degree is generically twice the motion polynomial degree but exceptional cases do exist. They are particularly interesting and allow geometric interpretation. Their study gives important insight into the structure of rational motions, where geometry and algebra are beautifully interwoven [2, 5].

MSC 2010: 70B10, 16Z05 15A66
Keywords: rational motion, motion polynomial, factorization, quaternion algebra, degree
Acknowledgement: Results presented in this talk have been obtained together with G. Hegedüs, Z. Li, D. F. Scharler, J. Schicho and J. Siegele. This research is supported by the Austrian Science Fund (FWF): P 31061 (The Algebra of Motions in 3-Space).

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On the Hyers-Ulam stability of some classical operators

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In 1940, Stanisław Marcin Ulam gave a talk before a Mathematical Colloquium at the University of Wisconsin in which he discoursed a number of important unsolved problems. Among those was a question concerning the stability of homomorphisms [1]. In 1941, Hyers published an article and gave an answer to the problem of Ulam for additive functions defined on Banach spaces [2]. In the following years this problem grew in popularity and many authors published articles on the stability of functional equations. As a result of the problem posed by Ulam, and afterwards the response Hyers gave to this problem, this terminology, which was included in many studies, was named "Hyers-Ulam Stability". This terminology has been continued and used for differential equations and various integral equations as well as functional equations.

In this talk, we will focus on the Hyers-Ulam stability terminology and refer about the Hyers-Ulam stability of some classical operators from approximation theory that have attracted considerable attention recently.

MSC 2010: 39B82, 41A35, 41A44
Keywords: Hyers-Ulam stability, positive linear operators

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A new approach to asymptotic integration of differential equations

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Asymptotic integration of differential equations is a classical research topic in applied mathematics. There is large body literature on the subject concerning a general class of second-order differential equations $x'' = f(t,x)$.The problem is to find conditions on the function $f(t,x)$ that guarantee the existence of solutions asymptotic to $a + bt$ as $t \to \infty$ for any given real numbers a and b.

In this talk, we will give a literature review, and then provide a new approach that can be used to study such problems for a class of more general differential equations. Our technique is different than the traditional ones used in the literature; it is a constructive approach applicable to various types of differential equations as well as difference equations, delay differential equations, and impulsive differential equations.

MSC 2010: 34A34
Keywords: Asymptotic integration, second-order, nonlinear, principal solution, nonprincipal solution

Abstracts of participants' talks

Odd automorphisms of the Grassmann algebra

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Consider the automorphism group $\mathrm{Aut}(G)$ of the Grassmann algebra G with a countable generating set X. In this work, we determine odd automorphisms sending each generator to elements expressed as a linear combinations of monomials of odd length. These automorphisms are of importance in describing the group $\mathrm{Aut}(G)$.

MSC 2010: 15A75, 14J50, 20F22
Keywords: Grassmann algebra, automorphism, subgroup

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Geodetic number of circulant graphs $C_n(\{1,3\})$

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The geodetic number of a graph is an important graph invariant. Geodetic number has many applications in Computer Networks, Social Networks. Let $1 \leq a_1 < a_2 < \cdots < a_m \leq \lfloor \frac{n}{2} \rfloor$, where m, n, a_i are integers, $1 \leq i \leq m$, and $n \geq 3$. Set $S = \{a_1, a_2, \cdots, a_m\}$. A graph G with the vertex set $\{1, 2, \cdots, n\}$ and the edge set $\{\{i,j\} : |ij| \equiv a_t(modn)$ for some $1 \leq t \leq m\}$ is called a circulant graph with respect to set S (or with connection set S), and denotes by $C_n(S)$ or $C_n(a_1, a_2, \cdots, a_m)$. In this paper, we compute the geodetic set and geodetic number of circulant graphs $C_n(\{1,3\})$.

MSC 2010: 05c99, 05cxx, 05Exx
Keywords: geodetic set; geodetic number; circulant graph

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On Toeplitz operators with Poly-quasihomogenous symbol

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In this paper, we give some basic results concerning Toeplitz operators whose symbol is of the form $f(\theta)\phi$, where ϕ is a radial function and $f(\theta)$ is a polynomial in $e^{i\theta}$, then use these results to characterize all Toeplitz operators which commute with them on the Bergman space.

Keywords: Toeplitz Operators, Bergman space, commuting problem, poly-quasihomogenous function

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Ev-degree and ve-degree molecular topological properties of silicon carbide structures

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Topological indices enable information about the underlying topology of chemical and physical substances. Topological indices are grouped into two categories; degree based and distance based. Degree based topological indices are defined by using classical degree concept in graph theory. Ev-degree and ve-degree which have been defined recently are two novel degree concepts in graph theory. Also Ev-degre and ve-degree based topological indices have been defined as parallel to corresponding classical degree based topological indices. Molecular classical degree based topological properties of silicon carbon structures have been investigated recently. In this study we investigate ev-degree and ve-degree molecular topological properties of two silicon carbon structures of $Si_2C_3 - I$ and $Si_2C_3 - II$.

MSC 2010: 05C07, 05C90
Keywords: Ev-degree topological indices, Ve-degree topological indices, Silicon carbide structures

Numerical simulation of two-dimensional sine-Gordon equation using the dual reciprocity boundary element method

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In this study two-dimensional sine-Gordon (SG) equation is solved by using the dual reciprocity boundary element method (DRBEM). At the beginning of the solution procedure, central difference approximations were used for both first and second order time derivatives. After inserting the finite difference approximations into the equation, the form of the modified Helmholtz equation was obtained. The fundamental solution of modified Helmholtz equation is employed in the integral equation formulation [1]. The inhomogeneous terms of the equation causes a domain integral in the boundary integral equation [2]. The DRBEM provides to transform the domain integral into the boundary integral by approximating with thin plate spline the inhomogeneous term of the eqution ($r^2 \ln r$). Numerical results are presented for several cases and they are observed to be in good agreement with other numerical results available in the literature [3].

MSC 2010: 65M38, 65N38, 35J05
Keywords: DRBEM, sine-GORDON, modified Helmholtz

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V_3 helices in the 5-dimensional Euclidean space

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In this paper, we give a definition of 3-type harmonic curvature functions of a regular curve. Henceforth, by using this, we obtain a characterization of V_3 helices in the 5-dimensional Euclidean space $\mathbb{E}^5$. Finally, we have an example of V_3 helix in $\mathbb{E}^5$.

MSC 2010: 53A04
Keywords: Harmonic curvatures, helix, Euclidean space

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Generalized spline and its algebraic structure

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Given a graph G whose edges are labeled by ideals of a commutative ring R with identity, a generalized spline is a vertex labeling by elements of R so that two adjacent vertices differ by an element of the ideal associated to the edge. The set of generalized splines form a subring of a product of R-copies and a module over R. Such a module it is called a generalized spline module. We focus on problems of the freeness, finding bases and bases criteria for generalized spline modules. We try to give an answer by using combinatorial and linear algebra techniques.

MSC 2010: 05C78, 11A07, 05C40, 13D40
Keywords: Generalized splines, algebraic graph theory, modules

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The numerical solutions of the time-fractional partial differential equations by natural transform decomposition method

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The natural transform decomposition method is applied to obtain the approximate solutions of the time-fractional partial differential equations with Caputo derivative. The fractional derivatives are defined in the Caputo sense. Numerical experiments are analyzed by NTDM. The graphs of the solutions of the some nonlinear time-fractional partial differential equations are plotted in the MAPLE software. The applications shows that NTDM is very effective method.

MSC 2010: 35F20, 35R11, 65R99
Keywords: Time-fractional partial differential equation, natural transform decomposition method, Mittag-Leffler function

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Condition number of a condition number for the structured matrices

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We can define a matrix function $f : \mathbb{K}^{n\times n} \to \mathbb{K}^{n\times n}$, ($\mathbb{K} = \mathbb{C}$, $\mathbb{R}$) as a mapping from a set of $n \times n$ matrices to a set of $n \times n$ matrices. It is crucial to understand the sensitivity of matrix functions to perturbations, which is measured by condition numbers (we call level-1 condition number). However condition numbers might not be computed exactly due to the errors in inputs. In this situation the problem of measuring the sensitivity of condition numbers emerges. The concept of the condition number of the condition number, named level-2 condition number, was first investigated by Demmel [1]. Higham evaluated the level-2 condition number for the matrix inverse and for the solution of linear systems for symmetric matrices [2]. For unstructured matrices, the relation between the level-2 condition number and the higher order Fréchet derivatives is given in [3].

The aim of this work is to investigate the structured level-2 condition number by following the similar steps for the structured level-1 condition number using differentials between smooth manifolds and enforcing the structure on the perturbation matrices [4]. We examine the effect of restricting the perturbation matrix to an automorphism group. This work also shows the necessity of structure preserving algorithms for the computation of matrix functions by comparing the unstructured level-2 condition number with the structured one for some specific matrix functions such as matrix square root, matrix exponential and matrix logarithm.

MSC 2010: 34B05, 34A08
Keywords: matrix function, Fréchet derivative, Kronecker form, level-2 condition number, matrix exponential, matrix logarithm, matrix square root, automorphism group

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N-dimensional φ-variation of nonlinear discrete operators

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In this work, we study the Tonelli sense N-dimensional φ-variation (see [1, 6, 7]) of the discrete operators studied in [2, 3]. First of all, we construct a nonlinear form of these operators. Then using Tonelli sense variation, we investigate the approximation properties of N-dimensional setting of the operators under φ-variational functional. Moreover, we apply summability method (see [4, 5]) in order to improve these type of operators to a general case. We also study the rate of convergence. Finally, using the relation between discrete operators and generalized sampling series, we also have convergence results in φ-variation for the generalized sampling series.

MSC 2010: 26A45, 40A25, 41A25
Keywords: φ-variation, Nonlinear operators, Discrete operators

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On generalizations of r-ideals of commutative rings

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In this study, we present generalizations of the concept of r-ideals in commutative rings with nonzero identity. Let R be a commutative ring with $0 \neq 1$ and $L(R)$ be the lattice of all ideals of R. In 2008, the authors introduced generalizations of prime ideals by defining the concept of ϕ-prime ideals, where $\phi : L(R) \to L(R) \cup \{\emptyset\}$ is a function, see [1]. On the other hand, in 2015, R. Mohamadian presented the notion of r-ideals in commutative rings with nonzero identity as followings: an ideal I of a commtative ring with identity R is called r-ideal (resp., pr-ideal), if whenever $ab \in I$ and a is regular element imply that $b \in I$ (resp., $b^n \in I$, for some natural number n), for each $a, b \in R$, see [3]. By the help of with these definitions, we introduce a concept of ϕ-r-ideals as followings: a proper ideal I of R is called a ϕ-r-ideal of R if whenever $ab \in I$ and $Ann(a) = (0)$ imply that $b \in I$ for each $a, b \in R$. In addition to giving many properties of ϕ-r-ideals, we also examine the concept of ϕ-r-ideals in trivial ring extension, see [2] and use them to characterize total quotient rings.

MSC 2010: 13A15, 13A18
Keywords: r-ideals, ϕ-prime ideals, ϕ-r-ideals

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A new paradigm on the stability of second order dynamical systems

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In this paper we are dealing with the physical implications of second order ordinary differential equations. In this connection we are deducing an obvious distinction between the qualitative behaviors of conservative and dissipative systems in the sense of Liapunov. With these intuitions some well-known published papers may need improvements.

The statistical convergence in Riesz spaces

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The statistical unbounded topological convergence was studied and investigated with respect to the solid topology in locally solid Riesz spaces (see, [1, 2]). Also, the statistical convergence on Riesz spaces was studied (see[3, 4]). In this paper, we introduce the statistical unbounded order convergence in Riesz spaces by developing topology-free techniques with the unbounded order convergence on Riesz spaces. Moreover, we give some relations with other kinds of convergences.

MSC 2010: 46A40, 10A05, 46B42
Keywords: Statistical *uo*-convergence, order convergence, order statical convergence, Riesz spaces

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An efficient numerical method for solving nonlinear singular boundary value problems arising in various physical models

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Due to singularity, it is not possible to obtain analytical solution for singular boundary value problem in general; moreover, classical numerical methods are not applicable. Therefore, various non-classical numerical methods proposed to solve this kind of problems. In this study, a useful technique, Chebyshev Finite Difference Method (CFDM) is introduced for solving nonlinear singular boundary value problems, since the present method is more advantageous for the solution of singular boundary problems than the other methods given in the literature. CFDM is widely used in the literature for the solution of second-order initial or boundary value problems [1-4]. Convergence and error analysis of the method are investigated. To show the applicability and efficiency of the presented method, the numerical solutions of four nonlinear singular boundary value problems which appear in applied science and engineering are given. Three of these examples arise in the study of thermal explosion (Example 1), steady-state oxygen diffusion in a spherical cell (Example 2) and the equilibrium of isothermal gas spheres (Example 3). The numerical results show that CFDM is rather efficient and accurate than the many methods given in the literature, without dividing the domain into the sub intervals. Besides, the proposed method does not need any modification for different boundary conditions.

MSC 2010: 34B16, 65L10, 65L12
Keywords: Chebyshev finite difference method, numerical solution, nonlinear singular boundary value problem

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Generalized Einstein tensor for an almost pseudo-Ricci symmetric manifold

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The object of the paper is to study the generalized Einstein tensor $G(X,Y)$ on almost pseudo-Ricci symmetric manifolds, $A(PRS)_n$. It is examined some cases of the generalized Einstein tensor on an $A(PRS)_n$ manifold and proved some related theorems. For instance, in an $A(PRS)_n$ manifold with non-zero scalar curvature, if the generalized Einstein tensor $G(X,Y)$ equals zero, then two associated $1-$forms are linearly dependent.

MSC 2010: 53B20, 53C15, 53C25
Keywords: Almost pseudo-Ricci symmetric manifold, Generalized Einstein tensor, Torqued vector field, Conservative, Cyclic paralel, Codazzi tensor

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Goodman and Kruskal's gamma coefficient for ordinalized bivariate distributions

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We consider a bivariate normal distribution with linear correlation ρ whose random components are discretized according to two assigned finite sets of thresholds. On the final bivariate ordinal random variable, one can compute Goodman and Kruskal's gamma coefficient γ, which is a common measure of ordinal association [1]. Given the known analytical monotonic relationship between Pearson's ρ and Kendall's rank correlation τ [2] for the bivariate normal distribution,

$$\tau = \frac{2}{\pi} \arcsin \rho, \tag{1}$$

and since Kendall's τ coincides with Goodman and Kruskal's γ for a bivariate continuous variable, the change of this association measure before and after discretization is worth being studied. We consider several experimental settings obtained by varying the two sets of thresholds, or, equivalently, the marginal distributions of the final ordinal variables; in particular, we examine the case of uniform, symmetrical unimodal or bimodal, and asymmetrical distributions, with different number of categories. This study, confirming previous findings, shows that the gamma coefficient is in absolute value always larger than Kendall's τ; this discrepancy shrinks when the number of categories increases or, holding the same number of categories fixed, by using uniform distributions. Based on these results, a proposal is suggested to build a bivariate ordinal variable with assigned margins and association (expressed in terms of Goodman and Kruskal's γ), by ordinalizing a bivariate normal distribution. Hints and advice are provided when discretization is carried out on other bivariate continuous distributions.

MSC 2010: 62H05, 62H20, 62E15
Keywords: bivariate normal distribution, contingency table, copula, discretization, latent variable, ordinal association

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Markov matrices on the set of polytopes

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This paper presents the set of Markov matrices is drawable. For the proof, we use the one-to-one corresponding among nxn Markov matrices, the solution of linear equation system from derived Markov property and the set of (n-1)-polytopes.

MSC 2010: 60J99, 52B70
Keywords: Convex polytope, geometry, manifold, Markov matrices

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Ulam type stability for higher order functional differential equation

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In this study, the Hyers-Ulam stability of some classes of functional differential equations was investigated by direct method and open mapping theorem. We think that the results in this paper will contribute to the related literature.
MSC 2010: 34K20, 34K30, 34D04
Keywords: Hyers-Ulam stability, higher order, open mapping theorem

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A new numerical approach for solving nonlinear Volterra integro differential equation

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In this article, we study quasilinear Volterra integro-differential equations (VIDEs). Asymptotic estimates are made for the solution of VIDE. Finite difference scheme which is accomplished by the method of integral identities with using of interpolating quadrature rules with weight functions and remainder term in integral form are presented for the VIDE. Error estimates are carried out according to the discrete maximum norm. The theoretical results are tested on numerical examples.

MSC 2010: 45J05, 65R20, 65N06
Keywords: Error estimate, finite difference method, Volterra-integro differential equation

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Using the curve fitting method to estimate the peak value of the COVID-19 pandemic with applications

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Mathematical modeling plays a major role in assessing, controlling, and forecasting potential outbreaks. In this study, the curve fitting method is taken into consideration. We give the method of the least squares as a standard approach in regression analysis that estimates the attainable maximum (peak value) of the Coronavirus infection that started in Wuhan, China, and spread to the world in a short time period. Finally, we demonstrated its applications for three countries and presented results clearly that earns further detailed disquisition.

MSC 2010: 65C20, 65D10, 62J05
Keywords: Mathematical model, COVID-19, coronavirus, curve fitting, the least squares method

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Numerical solution of the integro-differential equation containing boundary layer

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In this paper, we examine the singularly perturbed convection diffusion problem for a class of Fredholm integro-differential equation. We construct a fitted difference scheme on a piecewise uniform mesh and analyze the error estimates. We prove that the method is almost first-order convergent with respect to the perturbation parameter in the discrete maximum norm. Moreover, we present the numerical experiments which support the theoretical results.

MSC 2010: 65L10, 65L11, 65L12, 65L20, 65R20
Keywords: Fredholm integro-differential equation, boundary layer, finite difference method, uniform convergence

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A mathematical $SVEIR$ model considered the effect of different vaccination rate to subcompartments of susceptibles

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In this paper we introduce a vaccination model as mathematically in a population in which spread of a disease. This is a $SVEIR$ model but it has some different aspcets to the class S. We assume that the susceptible individuals consist of two separate subgroups: susceptible individuals with high risk and other susceptible individuals. Also these subgroups are vaccinated at different rates. This model considering the incubation period too, consists of a delay differential equation system.

We firstly present the equilibrium points and the reproduction number $\mathcal{R}_0$ which is a vital threshold in spread of diseases. Then we give some results about the local and global stabilities of the equilibrium points according that $\mathcal{R}_0$ is greater than one or not. To do these we use Lyapunov function and LaSalle Invariance Principle [1]. Finaly we present an example to show the effect of vaccination rate of high risk group to spread of the disease.

MSC 2020: 34D05, 34D08, 34D20, 92B05, 92D25, 92D30
Keywords: Stability analysis, Lyapunov function, mathematical epidemiology, basic reproduction number, vaccination effect
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A new interpretation of horizontal lifts on cotangent bundles

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Using the horizontal lift on tangent bundles, the horizontal lifts on cotangent bundles of some tensor fields are constructed with the aid of a musical isomorphism. Describing the gh-lift of tensor fields on cotangent bundle, a new interpretation of the horizotal lifts on cotangent bundles is obtained.

MSC 2010: 53A45, 55R10
Keywords: Horizontal lift, tensor fields, musical isomorphism, cotangent bundle, tangent bundle

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Some new inequalities for $n-$polynomial convex functions and applications

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In this paper, the author established a new identity for differentiable functions, afterwards he obtained some new inequalities for functions whose first derivatives in absolute value at certain powers are convex and $n-$polynomial convex. Also he given some applications for special means for arbitrary positive numbers.

MSC 2010: 26A15, 26D07, 26D08
Keywords: Convex function, Hadamard-like inequality, $n-$polynomial convex functions, improved power-mean inequality

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Some results on a second order conformable dynamic operator on time scales

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In this paper, we deal with a conformable dynamic operator of second order on an arbitrary time scale. We define an initial value problem consisting of a dynamic equation which is generated by the above-mentioned dynamic operator and initial conditions. We prove an existence and uniqueness theorem for the solutions of the initial value problem and we suggest a method to construct the solutions. Then, we prove the conformable Lagrange identity. After that, we derive a conformable boundary value problem which consists of a conformable dynamic operator of second order and boundary conditions. We prove Green's theorem with the help of the conformable Lagrange identity and we provide a characterization for the eigenvalues of the conformable boundary value problem. Presented results of this paper are generalizations of some results in [1] via conformable derivative.

MSC 2010: 34N05, 26A33, 34K08
Keywords: Time scales, conformable derivative, boundary value problems

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Similarity detection of algebraic plane curves using invariants

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A new algorithm is presented to detect whether two given rational algebraic plane curves are similar, by using differential invariants. At the same time, all such similarities, and all symmetries which are the particular case of equal curves are found. A complete theoretical explanation of all stages of the method is given. Also, the method has been implemented and tested in the MAPLE system for some curves.

MSC 2010: 53A55, 68W30, 14Q05
Keywords: Differential invariants, plane algebraic curves, similarity detection

Several relations on certain Hardy sums and two-term exponential sum

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In this paper, we deal with a computational problem of one kind mean value involving certain Hardy sums and the two-term exponential sum with the help of the properties of Gauss sums, and derive some interesting precise computational formulae.

MSC 2010: 11L40, 11F20
Keywords: Dedekind sum, Hardy sum, two-term exponential sums, mean value, computational formula

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New results on borderenergetic graphs

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The energy of a graph is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix [1]. In this study, we present the graphs whose energy is equivalent to a complete graphs energy and this kind of graphs are called borderenergetic graphs [2]. Borderenergetic graphs are widely studied in the literature, see [2, 3, 4]. We mainly consider the line graphs of regular and strongly regular graphs. Firstly, we obtain a condition for the line graph of disconnected regular graph consisting of p copies of connected regular integral graph and q copies of complete graph to be borderenergetic. Also we present a condition for the line graph of strongly regular graphs to be borderenergetic. In addition, we obtain the similar result for the complement of a strongly regular graph. Finally, we show by examples that there exist connected borderenergetic graphs, different from the complete graph.

MSC 2020: 05C50, 05C75, 05C76
Keywords: Borderenergetic graph, line graph, strongly regular graph
Acknowledgement: The second author is partially supported by TUBITAK and the Scientific Research Project Office (BAP) of Selcuk University.

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An application of improved Bernoulli sub-equation function method to the nonlinear conformable time-fractional equation

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In recent years, the fractional differential equations have become a useful tool for describing nonlinear phenomena of science and engineering models. Many of techniques have been applied to obtain the exact solutions of these equations.

In this work, we consider a nonlinear conformable time-fractional partial differential equation. We apply Improved Bernoulli Sub-Equation Function Method (IBSEFM) to the exact solutions. We construct the new exact solutions and plot the graphs of the solutions according to the parameters by the aid of mathematics software. The results show that this method is powerful mathematical tool to solve nonlinear conformable time-fractional equations arising in mathematical physics. Therefore, this method is very reliable, efficient and can be applied to the other nonlinear fractional differential models.

MSC 2010: 35C08, 34K20, 32W50
Keywords: Exact Solutions, conformable derivative, improved Bernoulli sub-equation function method

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Some characterizations on totally $\eta-$umbilical real hypersurfaces of a complex space form

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Some inequalities involving the Riemannian curvature invariants are investigated on totally η-umbilical real hypersurfaces of a complex space form. Obtained results are discussed for totally η-umbilical real hypersurfaces of the 6-dimensional complex space form.

MSC 2010: 53C15, 53C40, 53C42
Keywords: Complex space form, curvature, hypersurface

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An Artinian analogue of the Noetherian result on dimension of Noetherian modules

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In this study, we present a generalization of the theorem on the Krull dimension for Artinian modules over quasi-local rings (*i.e.*, rings with only one maximal ideal) to the case where the rings are not necessarily quasi-local.

Our main objective is to give an Artinian analogue of the following well-known Noetherian result.

Let R be a semi-local commutative Noetherian ring (where semi-local means R has only finitely many maximal ideals). Then, for any finitely generated R-module N, we have

$$\dim_R(N) = d(N) = \delta(N)$$

where $d(N)$ is the degree of the Hilbert polynomial associated to N, while $\delta(N)$ stands for the least number of elements $r_1, r_2, \cdots, r_n$, $n \in \mathbb{N}$, of R such that $\ell_R(N/(r_1, r_2, \cdots, r_n)N)$, the length of the R-module $N/(r_1, r_2, \cdots, r_n)N$, is finite. (See for example [3, p. 98]. We also refer to Chapter 4 of [1] for information about Hilbert polynomials.)

MSC 2010: 13E10, 13E05, 13A15
Keywords: Artinian modules, Noetherian modules, Ideal theory

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Lower bounds for blow up time of p-Laplacian equation with damping term

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In this presentation, we consider the p-Laplacian wave equation with damping term. There are lot of works on lower bounds for blow up time [1, 2, 3]. We will prove a lower bounds for blow up time under suitable conditions.

MSC 2010: 35B44, 35K55
Keywords: Lower bounds, p-Laplacian equation, blow up

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The proper class generated injectively by socle-free modules

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In this study, we introduced a new type of proper class, through Dickson torsion theory. A submodule U of a module V is called a semiartinian supplement (shortly, sa-supplement) submodule in V if there exists a submodule T of V such that $V = T + U$ and $U \cap T$ is semiartinian. A sequence $0 \to A \to^{f} M \to C \to 0$ is called SAS if $f(A)$ is an sa-supplement submodule of M. The class $\mathcal{SAS}$ of SAS sequences is a proper class which is coinjectively generated by the class of semiartinian modules (see [2, Theorem 3.1]). We show that (1) the proper class $\mathcal{SAS}$ is injectively generated by socle-free modules; (2)a ring R is right semiartinian if and only if every maximal right ideal of R has an sa-supplement in R . Moreover, we deal with modules which are sa-supplement in every containing module, namely sa-supplementing. Injective modules and semiartinian modules are obvious examples of sa-supplementing modules. It is shown that (1) an sa-supplementing module M is closed under sa-supplement quotients if and only if $M/sa(M)$ is injective; (2) a projective module P is sa-supplementing if and only if $P/sa(P)$ is a quotient of an injective module; (3)R is right semiartinian if and only if every right R-module is sa-supplementing; (4) R is right SSI ring, i.e. every semisimple right module is injective if and only if every right module is sa-supplementing. Furthermore, we studied modules whose all submodules are sa-supplement and modules whose all sa-supplement submodules are direct summand.

MSC 2010: 16D10, 18G25, 16D50
Keywords: Proper class of short exact sequences, sa-supplement submodule, sa-supplementing modules, semiartinian modules
Acknowledgement: This work was financially supported by the Scientific and Technological Research Council of Turkey (TUBITAK) (Project number: 119F176).

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Second order numerical method for the singularly perturbed Fredholm integro-differential problem with zeroth order reduced equation

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We study the convergence of the homogeneous finite difference method for singularly perturbed Fredholm integro-differential equation with two parameters. Numerical method based on using integral identities with exponential basis functions and interpolating quadrature rules with the weight and remainder terms in integral form. We establish second order uniform convergence on a special non-uniform mesh. Numerical results illustrating the efficiency of the method are presented.

MSC 2010: 45J05, 65L11, 65L12, 65L20
Keywords: Fredholm integro-differential equation, singular perturbation, finite difference method, uniform convergence

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Solution of Duffing equation with Fourier decomposition method

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In this article, we aim to find an alternative solution to the Duffing Equation, which is an important ordinary differential equation in mathematical physics. We used for solution Fourier Transform and Adomian decomposition method. Adomian polynomials have been used for nonlinear term. Finally we obtained the complete solution by using the Pade Approach from the approximate solutions founded with the Fourier Decomposition Method.

MSC 2010: 42A38, 34A34, 34B30
Keywords: Duffing equation, Fourier transform, Adomian decomposition method

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A numerical approach for solving singularly perturbed neutral type delay differential problem

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In this paper, we deal with singularly perturbed neutral type delay differential problem. We propose an efficient numerical approach for solving this problem. We analyze the precise knowledge about the behavior of the exact solutions of the problem and its derivative. Then, we construct a finite difference scheme on an appropriate piecewise Shishkin type mesh. It is prove that the method is almost first order convergent in discrete maximum norm with respect to the perturbation parameter. Furthermore, numerical results which support these theoretical results are presented.

MSC 2010: 34K40, 65L11, 65L12, 65L20, 65L70
Keywords: Singularly perturbed problem, neutral type, finite difference scheme, uniform convergence

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Reasons for students preparing for university examination willing to choose mathematics teaching

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The profession of mathematics is known to be difficult in society . However, mathematics is used in every field. In this direction, in this study, it is aimed to determine the factors that effect the choices of the students who are preparing for university exam and considering to choose mathematics teaching department. In this study, where qualitative research method was adopted, focus group interview was used as data collection technique. The aim of the focus group interviews is to gain in-depth, detailed and multidimensional qualitative information about the participants' perspectives, experiences, interests, experiences, tendencies, thoughts, perceptions, emotions, attitudes and habits on a given topic . In this context, the participants' perceptions and situations they want to attract attention in a suitable environment within the framework of the interviews are discussed. The participants were selected by purposeful sampling method among the students preparing for the university exam. In the study, as a data collection tool, firstly, some of the questions included in the scale of the reasons for choosing the teaching profession prepared by Ozsoy, Ozsoy, Ozkara and Memis (2010) were compiled and interview questions was prepared. Afterwards, the semi-structured interview form, which was created by finalizing the interview questions in line with the opinions of 3 professors who were experts in their fields, was used as the data collection tool of this study. As a result of the findings of the study, it was seen that the reasons for choosing mathematics teaching profession could be examined under seven headings as the reasons for choosing mathematics teaching profession are the effect of family and close environment, the effect of the teacher, the interest of this profession in web based examinations, the effect of job anxiety and guarantee, the effect of students' ideals, the effect of whether the profession is suitable for the cats and the needs of the education system and the desire to meet them. In addition, it was determined that some of the students who prepared for the exam did not have any negative thoughts about mathematics course even though they interpreted mathematics as a difficult course.

MSC 2010: 97C30, 97C80
Keywords: Mathematics teaching profession, student opinion, reasons for choosing a profession

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Analysis of the difficulties faced by 6th and 7th grade students in the circle sub-learning area teaching process through metaphor

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Mathematics is one of the basic courses of the curriculum that includes abstract elements. Geometry, on the other hand, constitutes the observable side of mathematics science, which is full of abstract concepts . However, there may be some difficulties due to many factors affecting the learning process. In this study, it is aimed to determine the difficulties faced by the 6th and 7th grade students in the teaching process of the circle sub-learning area with the help of metaphors. Metaphors are considered as a powerful mental tool that individuals can work to explain in understanding a highly abstract, complex or theoretical phenomenon (Cerit, 2008; Lakoff and Johnson, 2010). Phenomenology, which is one of the qualitative research methods, was used in the study. The data of the study was obtained by completing the sentences of When I learn the circle, which is the geometric shape, I find ... most difficult to do so. Because ... by 140 students studying in 6th (80 students) and 7th (60 students) grades. At the end of the study, it was seen that students developed metaphors in 7 different categories regarding the difficulties they faced during the circle teaching process. These categories are drawing circle with standard and open equations; spring length measurements; degrees and radians; internal, external, central and peripheral angle concepts; angles in triangular and rectangular regions within the circle; problems with shapes drawn in circles; tangent problems in circles. Among these categories, the most produced metaphor was found to be in the category of problems with shapes drawn in circles and tangential problems in circles. In addition, it was found that the 7th grade students developed more metaphors than the 6th grade students regarding the difficulties encountered during the circle teaching process.

MSC 2010: 97C30, 97C80
Keywords: Geometry, circle, difficulties encountered, metaphor, secondary school students

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Notes on the Hewitt realcompactification of an orbit space

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K. Srivastava [1] showed that when the action of a finite topological group on a Tychonoff space is given, the Stone-Čech compactification of the orbit space of the action is the orbit space of the Stone-Čech compactification of the space. We show that this statement holds also for the Hewitt realcompactification.

MSC 2010: 54D60, 54C45
Keywords: Realcompactification, orbit space, real maximal ideal

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On generalized covering spaces and monodromy groups

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In this talk, after reviewing concepts of generalized covering spaces and monodromy group $\pi(p, x_0)$ for any map $p : \widetilde{X} \to X$ with the unique path lifting property, we give some conditions under which generalized covering map $p : \widetilde{X} \to X$ is maximal.

MSC 2010: 57M05, 55M12
Keywords: Fundamental group, generalized covering spaces, monodromy groups

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On the exponential stability of delay neutral equations with periodic coefficients

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In this work, we investigate the exponentially stability of solutions of a class of nonlinear time-varying delay differential systems of neutral type with periodic coefficients. We obtain new sufficient conditions for the exponential decay of the solutions at infinity. We prove a theorem on the existence of the solutions. The technique of the proof is based on the direct Lyapunov method. Our result has contribution to the former and related literature on the topic of the paper.

MSC 2010: 34K20, 34K06, 34K40
Keywords: Neutral differential equations, exponential stability, time-varying delay

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Detecting affine equivalences of ruled algebraic surfaces using differential invariants

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Recently some authors investigated the symmetries, similarities and affine equivalences of ruled algebraic surfaces [1, 2]. In these studies, the authors construct some efficient algorithms that detect symmetries, similarities and affine equivalences of ruled algebraic surfaces. In this presentation, we investigate also the affine equivalences of ruled algebraic surfaces. However our approach is different. Firstly, we give affine differential invariants of curves. Then we reduce affine equivalence of surfaces to a new type of equivalence of curves forming the surfaces. Finally, we detect all affine equivalences of ruled algebraic surfaces.

MSC 2010: 53A55, 68W30, 14Q05
Keywords: Affine differential invariants, ruled surfaces, algebraic curves

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Variational iteration method for solving the linear 3D Schrödinger equation

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In this paper, the linear 3D time-dependent Schrödinger equation has been solved by using well-known He's variational iteration method. The results show that the variational iteration method is a powerful mathematical tool to solve Schrödinger equation with highly accuracy.

MSC 2010: 35A15, 35C05, 81Q05
Keywords: Three-dimensional Schrödinger equation, variational iteration method, variational theory, high accuracy

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Comparison of the performances of the fiducial-based tests using M and MML estimators in one-way ANOVA

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In this study, we propose a test based on M-estimators suggested by Huber [3] in the context of one-way ANOVA. While developing this test, fiducial approach proposed by Fisher [4, 5, 6] is utilized. In this approach, it is possible to obtain a probability distribution for the parameter of interest without any prior information by means of data generating equation. In the literature, there are very few studies developing tests by using the fiducial approach in the context of one-way ANOVA, see for example Li et al. [7] and Gven et al. [1] for details. Also, the performance of the proposed test is compared with the test based on Tiku's modified maximum likelihood (MML) estimators [2] via Monte Carlo simulation study, see Gven et al. [1]. An application of the proposed test is given at the end of the study.

MSC 2010: 62F03, 62F10, 62F35
Keywords: Fiducial Approach, One-Way ANOVA, M-Estimators, MML Estimators, Monte-Carlo Simulation

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A novel NTRU-like public key encryption scheme over group rings

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NTRU is a public-key encryption scheme which is based on quotient ring of polynomials [1]. The popularity of NTRU comes from its speed related to encryption and decryption processes and it also serves post-quantum cryptography while the classical public-key encryption schemes such as RSA and ECC are vulnarable in terms of security against quantum computers. There are many algebraic variants of NTRU such as GTRU, MaTRU, QTRU in [2], [3], [4] respectively. In this study, we construct a novel NTRU-like public-key cryptosystem based on polynomials over group rings.

MSC 2010: 81P94, 94A60, 94A62
Keywords: Group ring, NTRU, GTRU, MaTRU, QTRU

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Invariant differential operators of the group SO(3,2)

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Two different parameterization of the homogenous space on which $SO\,(3,2)$ group acts are considered. The infinitesimal operators of the quasi-regular representation of the group are obtained for these parameterizations. Using the infinitesimal operators, quadratic Casimir operators are given simultaneously.

MSC 2010: 22E60, 22E70, 20C33
Keywords: Lie group and algebra, Casimir operator, Representation theory

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Bifurcation analysis of a discrete population model with conformable fractional derivative

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In this study, we have considered a conformable fractional order differential equations with piecewise constant arguments for modeling a population model. First of all, we apply a discretization process to the model and obtain two dimensional discrete dynamical system. Secondly, necessary and sufficient stability conditions are determined by using the Schur Cohn criterion. In addition, bifurcation analysis show that the discrete system enters both Flip bifurcation and Neimark-Sacker bifurcation according to changing parameter r around the positive equilibrium point. All these mathematical results are interpreted biologically and provide a recipe for controlling the population. Finally, numerical simulations are used to demonstrate the accuracy of all theoretical results.

MSC 2010: 26A33, 34A08, 39A28, 39A30, 92B05
Keywords: Conformable fractional derivative, Piecewise constant arguments, Stability, Flip and Neimark-Sacker bifurcation

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Generalization of Hardy-Copson type inequalities to time scale nabla calculus

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In this talk several Hardy-Copson type dynamic inequalities are obtained via time scale nabla calculus. These inequalities are nabla counterparts of their delta versions and generalize and unify their continuous and discrete analogues.

MSC 2010: 34N05, 26D10, 26D15
Keywords: Time scale nabla calculus, Hardy inequality, Copson inequality

Mickens' type exact and nonstandard finite difference schemes for the generalized version of diffusion reaction equation

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We consider the following class of generalized diffusion reaction (GDR) equation:

$$D\phi_{xx} = \phi_t + k\phi\phi_x - \alpha w + \beta w^2, \quad 0 \leq x \leq 1, \ t \geq 0, \tag{1}$$

subject to certain initial and boundary conditions. Here $D > 0, \ k = -2D, \ \beta \neq 0$ and $\left\{\frac{\alpha}{\beta} \in \mathbb{R} \geq 0\right\}$. With the help of solitary wave solution of GDR equation, we develop the exact finite difference (EFD) scheme for (1). Furthermore a non-standard finite difference (NSFD) scheme is developed. The properties like positivity and boundedness is proved to be preserved by the proposed NSFD scheme. The scheme is shown to be stable, consistent and first-order accurate in both space and time. Approximate solutions of the GDR equation under given initial and boundary conditions are obtained using NSFD scheme and the maximum error of the computed solutions are calculated. The comparison has been made with other existing methods in order to depict the supremacy of our method. The scheme gives good accuracy even for few spatial division. Recently Namjoo et al. ([1]) have constructed the EFD and NSFD schemes for (1) with $\alpha = \beta$. The author has taken D, α as non-negative and k as positive. The authors have not discussed the stability of the NSFD scheme. Later Verma et. al ([1]) have constructed the EFD and NSFD schemes for the generalized Burger Fisher equation. Equation (1) is the generalized version of both [1] and [2, $\theta = 1$].

MSC 2010: 74S20, 65M06, 35K57, 35K61
Keywords: Non-standard finite difference scheme, stability, consistency, exact finite difference scheme, generalized diffusion reaction.
Acknowledgement: We are thankful to Prof. R.E. Mickens whose ideas are applied while constructing the EFD and NSFD schemes for GDR equation.

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On an initial and nonlocal integral boundary condition for a mixed type equation

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In this study, on an initial and boundary value problem for a mixed type equation is considered. A uniqueness theorem for the solvability of this problem is shown and constructed the solution as the sum of Fourier series. The stability of the solution with respect to initial function is proved.

In mathematical physics equations, nonlocal boundary conditions show that physical process not only at the point but also at the whole object. This type boundary conditions are examined in [1]-[4] and many other works for different mathematical physics equations.

MSC 2010: 35M11, 35M12, 35M13
Keywords: Mixed type partial differential equation, integral condition, initial and boundary conditions

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The number of m-nilpotent elements in nilpotent subsemigroup of Catalan monoid

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Let $\mathcal{C}_n$ be the Catalan monoid, that is, the semigroup of all order-preserving and order-decreasing full transformations of a finite chain, say $X_n = \{1, 2, \ldots, n\}$, let $N(\mathcal{C}_n)$ be the set of all nilpotent elements of $\mathcal{C}_n$. For each a of a finite semigroup S with zero 0, is called nilpotent if $a^m = 0$ for some $m \in \mathbb{Z}^+$, and furthermore, a is called m-nilpotent provided that $a^m = 0$ and $a, a^2, \ldots, a^m$ distinct. In this talk, we obtain a formula for the number of m-nilpotent elements, and so the number of m-potent elements in $N(\mathcal{C}_n)$ for $1 \leq m \leq n-1$.

MSC 2010: 20M20, 05A15
Keywords: Order-preserving, order-decreasing, nilpotent, m-nilpotent

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L^P- solutions integro-differential equations with singuler nonlinear kernels

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In this study, we focus on a variety of scalar integro-differential equations with singular nonlinear kernels including linear, nonlinear, and resolvent equations. The primary consequence contains a type of existence theorem which is associated with the fixed point mapping identified by the integro-differential equation and generates a unique solution having a continuous derivative in a very simple manner. Then we obtain qualitative properties of solutions by constructing a Liapunov functional.

MSC 2010: 34K05, 45M10
Keywords: Integro differential equations, Lyapunov functionals, L^P solutions

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On the Chebyshev-S polynomials and their matrix form

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The polynomials defined by the relation $S_n(x) = U_n\left(\frac{x}{2}\right)$ are called Chebyshev-S [1] such that $U_n(x)$ denote the Chebyshev polynomials of the second kind. The elements of $\{S_n(x)\}$ family are derived from using the recursive relation $S_n(x) = x.S_{n-1}(x) - S_{n-2}(x)$ with the initial conditions are $S_{-1}(x) = 0, S_0(x) = 1$ and $S_1(x) = x$ [1].

In this study, some of the applications given by W. Lang [2] related the Chebyshev-S polynomials has been introduced firstly. Then these polynomials are converted to a matrix form. In the system formed by thus, it has been focused on matrices which are coefficients matrix and inverse of coefficient matrix. The inverse matrix is associated with the Catalan numbers. It has been indicated that both matrices can be constructed with the help of the convolution of their columns. Moreover, whether both matrices are Riordan type matrix has been studied in this study.

MSC 2010: 11C08 , 11C20, 15B36
Keywords: Chebyshev polynomials, Catalan triangle, Pascal-like convolution matrix, Riordan type matrix

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G-radical units in commutative group rings

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Let R be a commutative ring with unity and G be an abelian group. RG and $V(RG)$ denote the group ring of G over R and the group of all normalized units in RG respectively. A unit $u \in V(RG)$ is said to be G-radical if $u = gw$ for some $g \in G$ and $w \in 1 + I(J(R)G; G)$ where $J(R)$ is the jacobson radical of R and $I(J(R)G; G)$ is a relative augmentation ideal of RG. In this study, we investigate necessary and sufficient conditions for

$$V(RG) = G \times (1 + I(J(R)G; G)).$$

MSC 2010: 16S34, 16U60, 20K10, 20K20, 20K21
Keywords: Group rings, units, radical, jacobson radical, commutative rings

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Krasnoselsky canonical domains, Lyapunov functions, and the existence of Poisson bounded solutions

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We introduce the concepts of Poisson boundedness and Poisson partial boundedness of solution of system of ordinary differential equations, which, unlike [1]-[5], do not require, respectively, the property of Poisson boundedness and Poisson partial boundedness for nearby solutions. On the basis of the Lyapunov vector function method and the Krasnoselsky method of canonical domains, we obtain sufficient conditions for the existence of Poisson bounded solutions and partially Poisson bounded solutions.

MSC 2010: 34C11 34D20 34A12
Keywords: Poisson boundedness of solution, partial boundedness of solution, existence of solutions, Lyapunov function, Krasnoselsky canonical domain
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Mother to child transmission of HIV/AIDS and its associated factors in Jigawa State Nigeria. A generalized linear spatial modelling approach

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Mother-to-child transmission of HIV is the spread of HIV from a woman living with HIV to her child during pregnancy, breastfeeding and at delivery. The aim of this study was to investigate mother-to-child transmission of HIV/AIDS infection and its associated factors in Jigawa State, Nigeria using Generalized Linear Spatial Model (GLSM) approach. The specific objectives of the study was to assess influence of home deliver on mother to child transmission of HIV to evaluate the extent to which model of breast feeding affects mother-to-child transmission of HIV infection. A secondary data was collected from the ART unit of five general hospitals and one specialist hospital of Jigawa State. A generalized linear spatial model was used to estimate the model parameters using Poisson Informative Prior, Multivariate Normal Prior, Independent Cauchy Prior. The analysis result revealed that mother living with HIV and children who were infected though MTCT of HIV considered in the analysis are influenced by model of delivery, mode of breast feeding, number of ANC visit, mothers knowledge on HIV status and mother age which were found to be the significant determinants f mother-to-child transmission of HIV/AIDs in Jigawa State. Therefore, this study recommended that, more prevention campaign should be planned and carried out among pregnant women living with HIV/AIDs while the monitoring system of MTCT of HIV/AIDs in Jigawa State require improvement in terms of data complement and integration to allow for better assessment of the epidemic.

MSC 2010: 91B72, 91D25, 62H11
Keywords: Mother-to-child Transmission, generalized linear spatial model (GLSM), HIV, Jigawa, Nigeria

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Natural mates of Frenet curves in three-dimensional Lie group

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In this study, we introduce the natural mate of a Frenet curve in a three dimensional Lie group $\mathbb{G}$ with bi-invariant metric. Also, we give some relationships between a Frenet curve and its natural mate in $\mathbb{G}$. Especially, we obtain some results for the natural mate of a Frenet curve in $\mathbb{G}$ when the Frenet curve is a general helix, a slant helix, a spherical curve, a rectifying curve, a Salkowski (constant curvature and non-constant torsion), anti-Salkowski (non-constant curvature and constant torsion).

MSC 2010: 53A04, 53A35, 22E15
Keywords: Natural mate, conjugate mate, helix, slant helix, spherical curve, rectifying curve, Salkowski curve, anti-Salkowski curve

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A note on the Ψ-stability of solutions of Volterra integro-differential equations

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The purpose of this paper is to discuss Ψ-stability properties of solutions of some systems of Volterra integro-differential equations. Here, Ψ-stability properties of solutions are investigated by the comparison principle and based on the definitions of the Lyapunov stability and its generalizations. Moreover, some numerical examples are given for the illustrations.

MSC 2010: 34A30, 34C11, 34D05, 34D20
Keywords: Lyapunov stability; Ψ-stability; system of Volterra integrodifferential equations

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On solving nonlinear Fredholm integral equation with optimization

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In this work optimization methods used to solve nonlinear Fredholm integral equation of the second kind. This is achieved first by substituting the integral part by a quadrature method then the problem is modified to an unconstraint optimization problem. Some examples are given.

MSC 2010: 45B05, 49M15
Keywords: Fredholm integral equation, unconstraint optimization, numerical integration

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Unsteady heat and mass transfer magnetohydrodynamic (MHD) convective Couette flow with thermal radiation using finite element method (FEM)

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In this study, unsteady hydromagnetic convective Couette flow of a viscous, electrically conducting and incompressible fluid taking into consideration thermal radiation, diffusion and diffusion thermo effects is conducted numerically employing Finite Element Method (FEM). Computations were done for a wide range parameters of engineering interest embedded in the fluid flow problem. The effects of these flow parameters on the velocity, temperature and concentration are presented graphically while that of the skin-friction, Nusselt and Sherwood numbers are gotten and displayed in tabular forms. The obtained results are benchmarked with previously published works and was seen to be in good agreement.

MSC 2010: 65M38, 65N38, 35J05
Keywords: Magnetohydrodynamic (MHD), Free convection, FEM, Heat and mass transfer

Learning difficulties in limit primary mathematics teachers candidates

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This research is a qualitative study that examines the learning difficulties related to the limit, given in the Calculus 1 course of elementary mathematics teacher candidates.The research was carried out with 24 pre-service teachers selected from 90 pre-service teachers studying in the first grade of Primary Mathematics Teaching at a university and attending Calculus 1 course in 2019-2020 academic year. In this research, after the lectures of the lecturer the following questions are asked to students every week: What are the concepts and reasons you understand easily? What are the concepts and reasons that you understand hardly? If you tell the concepts given in the course of Calculus 1 to your friend, how would you tell? For the purpose of the research, the data obtained from the answers given by prospective pre-service teachers for 3 questions were analysed with content analysis. As a result of the content analysis, the sub-themes were created, which included 3 main themes, namely Epistemological Causes, Pedagogical Causes and Psychological Causes, as the learning difficulties of prospective pre-service teachers. According to the findings obtained from the research, sub-themes were obtained in the form of epistemological causes under the main theme, not to understand the definition of limit, excessive use of symbolic representation, not understanding the concept of pierced neighbourhood, lack of prior knowledge and not being able to detail the subject. Under the main theme of pedagogical reasons, sub-themes were obtained such that the lecturer explained the subject quickly, the lecturer had a different form of expression than the previous teachers seen in the high school. Under the main theme of psychological causes, sub-themes were obtained such as the physical conditions of the class, lack of attention (lack of focus), the student's bias towards the lesson and the need for more lesson time.

MSC 2010: 40A25, 97B50, 97D70
Keywords: Limit, Learning Difficulties, pre serviece mathematics teacher

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Regularity conditions for endomorphism rings of Leavitt path algebras

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Let E be any graph, K be any field. The Leavitt path algebras of E with coefficients in K denoted by $L := L_K(E)$ were introduced in [1] and then extended to arbitrary graphs in [2]. In [3], Abrams and Rangaswamy showed what regularity conditions of Leavitt path algebras are.

In this talk, we will give introduce endomorphism rings of Leavitt path algebras as a right module L. Furtermore, we will give some regularity conditions of endomorphism ring of Leavitt path algebras.

MSC 2010: 16D50, 16E50, 16W20
Keywords: Leavitt path algebras, endomorphism rings, von Neumann regular rings, locally regular rings

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Blow up of solutions for a logarithmic quasilinear hyperbolic equation with delay term

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In this work, we investigate a logarithmic quasilinear hyperbolic equation with a delay term in a bounded domain. Under suitable conditions, we obtain the blow up of solutions in finite time. Time delays arise in many applications, for instance, physical, chemical, biological, thermal and economic phenomena.

MSC 2010: 35L05, 35B44
Keywords: Blow up, quasilinear hyperbolic equation, delay term

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Rate of approximation for modified Lupaş-Jain-Beta operators

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The main intent of this paper is to innovate a new construction of modified Lupaş-Jain operators with weights of some Beta basis functions whose construction depends on σ such that $\sigma(0) = 0$ and $\inf_{x\in[0,\infty)} \sigma'(x) \geq 1$. Primarily, for the sequence of operators, the convergence is discussed for functions belong to weighted spaces. Further, to prove pointwise convergence Voronovskaya type theorem is taken into consideration. Finally, quantitative estimates for the local approximation are discussed.

MSC 2010: 41A10, 41A25, 41A36
Keywords: Generalized Jain operators, Beta basis functions, Korovkin's type theorem, Voronovskaya type theorem

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Analysis of mean time to failure of four series parallel systems with mixed standby units

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The present paper studies and compare the mean time to failure (MTTF) of seriesparallel systems containing five units each. Four different configurations are considered in this paper. It is assumed that failure and repair time of all units are assumed to be exponentially distributed. Explicit expressions for mean time to failure are derived, examined and compared. Analytical and numerical computations are presented to compare their mean time to failure. Cost/benefit measure have been obtained for all configurations, where the benefit is mean time to failure. From cost benefit analysis, analytical and numerical results, the optimal configuration is configuration II.

MSC 2010: 90B25, 68N30
Keywords: Mean time to failure, mixed standby, series parallel

Detecting equivalences of hypersurfaces using differential invariants

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Let $\{g_{i,j}\}_{i,j=1}^{n}$ and $\{L_{i,j}\}_{i,j=1}^{n}$ be the sets of all coefficients of the first and second fundamental forms of a hypersurface x in $\mathbb{R}^{n+1}$. For a connected open subset $U \cup \mathbb{R}^n$ and a C^∞-mapping $x : U \to \mathbb{R}^n$ the hypersurface x is said to be d-nondegenerate, where $d \in \{1, 2, ..., n\}$, if $L_{dd}(x) \neq 0$ for all $u \in U$. Let $M(n) = \{F : \mathbb{R}^n \to \mathbb{R}^n \mid Fx = gx + b, g \in O(n), b \in \mathbb{R}^n\}$, where $O(n)$ is the group of all real orthogonal $n \times n$-matrices, and $SM(n) = \{Fx = gx + b \in M(n) \mid g \in SO(n)\}$, where $SO(n) = \{g \in O(n) \mid det(g) = 1\}$. In the present study, it is proved that the set $\{g_{ij}(x), L_{dj}(x); i, j = 1, 2, ..., n\}$ is a complete system of the $SM(n+1)$-invariants of a d-nondegenerate hypersurface in $\mathbb{R}^{n+1}$. A similar result has obtained for the group $M(n+1)$.

MSC 2010: 53A55, 53A07, 14J70
Keywords: Hypersurface, Bonnet's theorem, differential invariants

Dynamic implications of persistence in status concern

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This paper extends the models of *status concern* (*absolute wealth effect*) incorporating a memory effect. Infinitely lived representative household takes utility from her current as well as her past wealth levels, i.e. $\tilde{k}(t) = \tilde{k}_0 e^{-at} + a\int_0^t k(\tau)e^{-a(t-\tau)}d\tau$. In this study, stability properties and transitional dynamics have been assessed. As is typical with the wealth (see [1] and [2] among others) in utility literature, wealth effect causes over-accumulation of capital. Over-accumulation increases as the density of the wealth in utility increases. Memory can drive the economy into short run fluctuations. It is shown that as the memory on the past status weakens, steady state level of capital as well as steady state consumption increases. Moreover, we show that as the economy gets more industrialized, the effects of the status concern and its persistence on the steady state level of capital weakens.

MSC 2010: 49L20, 34K35, 34H05
Keywords: Optimal growth model, wealth effects, distributed delay, optimal control of integro-differential equations

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Maclaurin coefficient bounds for a subclass of bi-univalent functions associated with k-analogue of Bessel function

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The theory of q-calculus plays an important role in many areas of mathematical, physical and engineering sciences. Jackson (see[4] and [5]) was the first to have some applications of the q-calculus and introduced the q-analogue of the classical derivative and integral operators [1]. Later, Srivastava and Bansal (see [3], pp. 62) used the q-analogue of derivative in Geometric function theory by introducing the q-generalization of starlike functions for the first time, see also [2], pp. 347 et seq.

In this presentation, using by the concept of q-calculus theory, we firstly present a new subclass of the bi-univalent functions class δ associated with k-analouge of Bessel function in the unit disc Λ. Moreover, we derive initial two coefficient estimates belong to this subclass. We also give some relevant consequence of our main theorem.

MSC 2010: 30C45, 30C50
Keywords: k-analouge of Bessel function, bi-univalent functions, coefficient bounds

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Freeness of generalized spline modules over polynomial rings

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Given a graph G whose edges are labeled by ideals of a commutative ring R with identity, a generalized spline is a vertex labeling by the elements of R such that the difference of the labels on adjacent vertices lies in the ideal associated to the edge. The set of generalized splines has a ring and an R-module structure.

In this talk we focus on the freeness of generalized spline modules in case of R is multivariate polynomial ring. We first introduce the syzygy module on the cycles of an edge labeled graph and the edge decomposition operation. We give some results related to the freeness of generalized spline modules via projective dimension of the module.

MSC 2010: 05C78, 13C10, 13F20
Keywords: Generalized splines, Polynomial rings, Projective dimension

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Finite element solution of MHD flow in a T-Junction

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In this work, numerical solution of the magnetohydrodynamic flow in a T-junction with 90° angle, the junction has two inlet and only one outlet, is acquired using finite element method. In the T-junction, the flow is organized as a combining flow like in [2]. As a solution procedure, we use the finite element method (FEM) both the reciprocal convention diffusion type MHD equations considering no-slip boundary conditions. The obtained numerical solutions displayed in terms of figures for the several values of the Hartmann number.

MSC 2010: 76W05, 74S05, 65N30
Keywords: MHD Flow, T-Junction, FEM

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On contra we^*-continuous functions

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One of the most important concepts of mathematics is undoubtedly the concept of continuity. Recently, weak and strong forms of the concept of continuity have been studied by many mathematicians. Both the characterizations and some basic properties of these concepts have been investigated. Some of these are w-continuity [1], contra continuity [2] and contra $w\beta$-continuity [5]. In this study, the concept of contra we^* -continuity, which is weaker than the concept of contra $w\beta$-continuity, is introduced and some characterizations of this concept are obtained. Also, some basic features of the concept of contra we^* -continuity have been investigated. The relations of this concept with other types of continuity in the literature are revealed.

MSC 2010: 54C05, 54C08, 54C10
Keywords: we^*-open set, we^*-closed set, contra we^*-continuity

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On the automorphisms of generalized algebraic geometry codes

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In this work, we have considered generalized algebraic geometry codes that formed by two collections of places with each places in one group have same degrees. We introduce a new specific subgroup of the automorphism group of an GAG code. We show that a certain subgroup of $Aut(F/\mathbb{F}_q)$ can be embedded into this subgroup on some conditions. Then we show that when corresponding function field $F/\mathbb{F}_q$ is rational, this two subgroup can be isomorphic.

MSC 2010: 94B27, 14H37, 14H05
Keywords: Geometric Goppa codes, generalized algebraic geometry codes, code automorphisms, automorphism groups of function fields

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Similar and self-similar null Cartan curves in Minkowski-Lorentzian spaces

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In this paper, we study the similarity curvatures of null Cartan curves in the Minkowski (n+2)-Spaces. We investigate the fundamental theorem for a null Cartan curve in similarity geometry and we give the characterization of all self-similar null Cartan curves parameterized by pseudo-de Sitter parameter, which are null Cartan curves with constant curvatures in similarity geometry, in Minkowski space-time, de Sitter 3-space and hyperbolic 3-space.

MSC 2010: 53A35, 53A55, 53B30.
Keywords: p-similarity, null Cartan curve, self-similar, Cartan curvatures

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Hybrid fixed point theorems for the multi-valued operators in WC-Banach algebras

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We give the multi-valued versions of hybrid fixed points of nonlinear operators in WC-Banach algebras. We also present the existence and uniqueness of hybrid fixed points of Krasnoselskii-type nonlinear inclusion of the form

$$u \in Pu + QuRu, \quad u \in U$$

in WC-Banach algebra A. Where U is a nonempty weakly closed convex subset of A, P is a linear multi-valued operator and R is a nonlinear multi-valued operator.

MSC 2010: 34K13, 47H04, 47H10
Keywords: Krasnoselskii fixed point theorem, weakly sequentially upper-semicontinuous operator, linear multi-valued operator

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Some numerical results on the software HANSO

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Some numerical results obtained by using the software HANSO (Hybrid Algorithm for Non-Smooth Optimization) will be presented in this presentation. During the presentation, one can find all the details about the software HANSO by explaining how it works and which algorithms have been used. For these algorithms, a review will be given with details. In addition to this review, the characteristic of the test problems which are used for numerical experience will be also given briefly.

MSC 2010: 65K10, 90C90, 47X04
Keywords: Non-smooth optimization, BFGS, GSA

Evaluating the factors affecting success of students with the intuitionistic fuzzy PROMETHEE method

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In this study, an application of the PROMETHEE method, which is one of the multi criteria decision making methods, has been made in the field of education. The aim of this study; it is to create an algorithm that takes into account not only exam scores but also other factors affecting success of students. According to this algorithm, the factors affecting student achievement and exam scores have been evaluated together and the evaluation scores of the students have been calculated with the intuitionistic fuzzy PROMETHEE method. Thanks to this system that we will create, which will have the ability to apply not only in the field of education, but also in many fields, many factors will be evaluated simultaneously and different degrees of importance could be determined for each factor.

MSC 2010: 90B50, 03E72
Keywords: Multi criteria decision making, intuitionistic fuzzy sets, PROMETHEE method, success of students
Acknowledgement: Feride Tuğrul has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK 2211-A Domestic PhD Scholarship Program).

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On the qualitative properties of solutions of certain neutral equations

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Here, a nonlinear neutral differential equation with three variable delays is considered. A stability theorem is verified for the solutions of the considered equation. The technique of the proof is based on the Banach contraction mapping principle. We aim to contribute the literature. Our result includes and improves some former results on the stability of the solutions of delayed neutral differential equations, which are available in the literature. An example is provided to illustrate the proposed theorem. Our aim is to do a contribution to the qualitative theory of solutions.

MSC 2010: 34K20, 34D20
Keywords: Stability, Banach contraction mapping principle, (neutral) integro-differential equation, variable delay

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On the qualitative criteria for integro-differential equations with Caputo fractional derivative and constant delays

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The aim of paper is to investigate qualitative properties of solutions such as uniformly stability, asymptotically stability and boundedness of a non-linear Volterra integro -differential equations with Caputo fractional derivative and delays. To reach the aim of this paper, an appropriate Lyapunov function is defined, then by help of this function and the Lyapunov- Razumikhin method, some theorems, which include sufficient conditions, are proved on the mentioned concepts. As application, an example is provided to illustrate the proposed theorems. The obtained results are new, original and they have contributions to the topic of the paper and literature.

MSC 2010: 34D05, 34K20, 45J05
Keywords: Lyapunov- Razumikhin method, Lyapunov function, Volterra integrodifferential equation, stability, boundedness, integrability

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On the qualitative analysis of solutions of Volterra integro-differential equations with infinite delay

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We deal with a nonlinear Volterra integro-differential equation with infinite delay (IDDE). We prove three new results, which have sufficient conditions, on stability, uniformly stability and integrability of solutions of that IDDE. The proofs of the results are proceeded by Lyapunov-Krasovskii functionals. Our results extend and improve some results available in literature. We also give some example to illustrate the results.

MSC 2010: 45J05, 34K05, 34K20
Keywords: Nonlinear, Lyapunov-Krasovskii functional, Volterra, stability, uniformly stability, square integrability

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An approximate method for a delay differential equation with layer behavior

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In this study, we consider the singularly perturbed initial-value problem for a class of second-order delay differential equation. We construct a fitted difference scheme on a uniform mesh and obtain first-order uniform convergence with respect to the perturbation parameter in the discrete maximum norm for the method. We also present the numerical results that support the theoretical results.

MSC 2010: 34K06, 65L05, 65L11, 65L12, 65L20
Keywords: Delay differential equation, initial layer, finite difference method, uniform convergence

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Well ordered monotone iterative technique for a class of 2^{nd} order nonlinear 4-point Neumann BVPs

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In this article we have explored Monotone Iterative Technique for a class of nonlinear four-point Neumann BVPs

$$\begin{aligned} -x''(t) &= h(t, x, x'), \\ x'(0) = \lambda x'(\alpha), \quad x'(1) &= \delta x'(\beta), \end{aligned} \tag{1}$$

where $0 < \lambda$, $\delta < 1$, $0 < \alpha \leq \beta < 1$. The nonlinear term $h(t, x, x') : \Omega \to \mathbb{R}$ is Lipschitz in $x'(t)$ with Lipschitz contstant $K_2(t) \geq 0$, where $\Omega = [0, 1] \times \mathbb{R}^2$. To study the existence of solution of (1) we develop Monotone Iterative (MI) Technique, based on quasilinearization, in the presence of well ordered upper and lower solutions. To construct monotone sequences of upper and lower solutions with initial guesses we prove Maximum principle. Then under certain assumptions we prove that these sequences converges uniformly to the solution $x(t)$ of (1) in the specific region, where $\frac{dh}{dx} < 0$. This article is extension of [1], where $\lambda = 0$. MI-technique is also discussed by Verma et. al. [2] for BVPs (1) with boundary conditions $x'(0) = 0$ and $x(1) = \lambda x(\alpha) + \delta x(\beta)$. To demonstrate that the proposed technique is quite effective we compute the solution of the nonlinear multipoint BVPs which may not be computed easily in closed form. We have used Mathematica 11.3 to verify the conditions and convergence of monotonic sequences.

MSC 2010: 34L30, 34B27, 34B15.
Keywords: Monotone iterative technique, Well ordered upper and lower solutions, Four-point nonlinear ODE, Green's function, Maximum principle.
Acknowledgement: I would like to thank to my supervisor Dr. Amit K. Verma. This work is totally supported by him.

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Trans_maxima; A Maxima package for analytical solution of one dimensional linear system of transport equations

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In this study we aim to develop a Maxima [1] package for analytical solution of linear system of one dimensional transport equations of the form

$$U_t + AU_x = BU_{xx} + CU + D \tag{1}$$

where $U = \begin{bmatrix} u \\ v \end{bmatrix}$ is the vector of unknowns, A, B and C are 2×2 constant matrices and $D_{2\times1}$ is a constant vector. The package aims to solve the system over a finite interval with Dirichlet and Neumann boundary conditions, as well as Robin conditions with the aid of some numerical methods that may be needed for numerical computation of eigenvalues. The package aims to interactively asks for problem data (the coefficient matrices, initial and boundary conditions) and display both analytical solution and its graph if desired. Initial results for scalar transport equation will be illustrated.

The motivation for the study came from a research work on wound healing model being studied by the authors.

MSC 2010: 35E05, 35E20, 35Q92, 35G35, 68U01
Keywords: Transport equation, analytic solution of partial differential equations, Maxima packages

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On Z-symmetric manifold admitting projective curvature tensor

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The object of the present paper is to study the Z-symmetric manifold with projective curvature tensor. In this paper, we prove some theorems about these manifolds by using the properties of the Z-tensor. The existence of such notion is ensured by a non-trivial example.

MSC 2010: 53B20, 53C15, 53C25
Keywords: Codazzi tensor, harmonic function, recurrent tensor

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On the periodic solutions for a class of nonlinear n-th order differential equation with mixed delays

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In this paper, we use the coincidence degree theory to establish new results on the existence of T-periodic solutions for a class of nonlinear n-th order differential equations with mixed delays, which extend some previously known researches. Moreover, an example is given to illustrate the feasibility and application of the achieved results.

MSC 2010: 34C25, 34D40
Keywords: Periodic solutions, mixed delays, coincidence degree theory

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Maps corresponding to $\Gamma_0(N)$ for a non-transitive action of the normalizer

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Recently, the authors in [1] investigated the maps corrsponding to $\Gamma_0(N)$ where the normalizer of $\Gamma_0(N)$ in $PSL(2,\mathbb{R})$ is a triangle group. In their study, the normalizer acts transitively on the set of the extended rational numbers $\hat{\mathbb{Q}} = \mathbb{Q} \cup \{\infty\}$.

In this study, we investigate the maps corresponding to $\Gamma_0(N)$ for a non-transitive action of the normalizer and N will denote a natural number of the form 2^3p^2, where $p > 3$ is a prime. We start by defining a universal map arising from the non-transitive action of the normalizer. Then we construct the maps corresponding to $\Gamma_0(N)$ using the universal map. Once the maps are constructed, their vertices, edges, axes and faces will be obtained. One can see that the all maps obtained in this way will be quadrilateral.

MSC 2010: 05C10, 20H10, 57M60
Keywords: Regular maps, Modular group, Normalizer

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Some remarks on triangular norms on bounded lattices

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Triangular norms and dually triangular conorms were firstly defined on the unit real interval [1,2] and extensively investigated in [3,4,7]. Afterward, triangular norms were defined on bounded lattices and studied in many respects [5,6,8].

In this paper, we discuss triangular norms on bounded lattices. Some construction methods are introduced and illustrated by examples. In addition, we show its difference from the current methods in the literature.

MSC 2010: 03E72, 03B52, 03G10, 18B35
Keywords: Triangular norms, bounded lattices, construction methods

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On the asymptotic stability of a nonlinear system with multiple delays

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This paper deals with delay-dependent stability of zero solution a nonlinear differential equation system with multiple time-varying delays. Based on the Jensen inequality, the Wirtinger inequality and the Lyapunov-Krasovskii functional approach, some new sufficient conditions are derived on the asymptotic stability of zero solution of the considered system. In addition, two numerical examples are given to show the applicability of the derived results by MATLAB-Simulink.

MSC 2010: 34K20, 34K06, 34K12
Keywords: Asymptotic stability, Lyapunov-Krasovskii functional, Jensen inequality, multiple delays, Wirtinger inequality

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On discrete convex optimization of interval valued functions

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In this paper, we study a class of convex type interval-valued functions on the domain of the product of discrete subsets of real numbers. By considering LW order relation on the class of closed intervals, we proposed some optimal solutions. LW convexity concepts and generalized Hukuhara differentiability (viz. delta and nabla) for interval-valued functions yield the necessary and sufficient conditions for an interval programming problem. Also, we compare our results with the results given in the literature. These results may open a new avenue for modeling and solve different types of optimization problems that involve discrete variables.

MSC 2010: 90C29, 90C30
Keywords: Interval-valued functions, delta-differentiability, KKT-optimality conditions

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