

Rational Kinematics

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A rigid body motion is called “rational” if all trajectories are rational curves [4]. Rational motions have been investigated since the second half of the 19th century but, nonetheless, some of their rather fundamental properties have been unveiled only in the past decade. In our talk we present an overview of these results, accessible for a general mathematical audience.

A central aspect is the parametrization of rational motion via certain polynomials over rings (“motion polynomials”). Factorization of motion polynomials [1, 3] corresponds to the decomposition of rational motions into sequences of rotations and has important applications in mechanism science. The trajectories’ degree is generically twice the motion polynomial degree but exceptional cases do exist. They are particularly interesting and allow geometric interpretation. Their study gives important insight into the structure of rational motions, where geometry and algebra are beautifully interwoven [2, 5].

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