On Hardy, Copson, Bennett, Leindler type dynamic inequalities

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Hardy's celebrated inequalities [1, 2] in both discrete and continuous forms gave inspirations to many mathematicians such as Copson, Bennett, Leindler and so on. They generalized and improved Hardy's inequalities and their reverse versions, which are called Bennett-Leindler inequalities, for both cases seperately. In order to avoid proving results twice, once for continuous functions and once for functions defined on discrete sets, time scale unification of these inequalities have appeared in the literature by using delta and nabla derivatives and integrals. Since these unifications are not adequate in the theoretical research of some differential and difference equations and in certain computational applications such as adaptive computing and multiscale methods [3], the diamond alpha, $\diamond_\alpha$, calculus, which utilizes convex linear combinations of delta and nabla derivatives and integrals, has been introduced by Sheng et al. [3].

In this talk we give a survey for Hardy, Copson, Bennett and Leindler type dynamic inequalities on time scale delta and nabla calculii. Then we present two kinds of dynamic Bennett-Leindler type inequalities via the diamond alpha calculus. The former, which is composed of inequalities of mixed type containing delta, nabla and diamond alpha integrals together, strengthens and binds the existing results obtained for time scale delta and nabla calculii. The latter, which consists of inequalities including only diamond alpha integrals, harmonizes and unifies the foregoing results. Moreover these inequalities provide novel and better results in the special cases.

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