On the existence and stability of symmetric solutions in a class of weakly non-linear systems

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For a class of systems of weakly non-linear ordinary differential equations

$$u'(t) = \varepsilon h(t, u(t)), \qquad t \in J,$$

where $J \subset \mathbb{R}$ is an open interval and ε is a small parameter, we study solutions satisfying the condition

$$u(\psi(t)) = Au(t), \qquad t \in J,$$

which, with different choices of the function $\psi : J \to J$ and the matrix A, includes various frequently encountered properties (e.g., oddness, periodicity, antiperiodicity or affine periodicity). It is assumed that the continuous function h is smooth in the space variable and satisfies a suitable invariance condition.

For solutions having the property mentioned, effective existence and asymptotic stability conditions are obtained, which are closely related to classical results. The asymptotic stability conditions are formulated in terms of either logarithmic norms or spectral abscissas of certain matrices that can be found explicitly.

We review the main results, outline the ideas of proof, and mention the relation to known statements.

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References

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