Stability of infinite dimensional systems subject to perturbations.

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We investigate the ISS property of two nonlinear dynamical systems with infinite dimensional state spaces.

Problem 1. Let consider the following system of differential equations

$$\begin{aligned} u_t(z,t) &= \mathcal{A}u(z,t) + B(z,t)x(t), \\ \dot{x}(t) &= C(t)x(t) + \int_0^l D(z,t)u(z,t)\,dz, \end{aligned} \tag{1}$$

with initial

$$x(0) = x_0 \in \mathbb{R}^n, \quad u(z,0) = \varphi(z), \quad \varphi \in L_r^2([0,l]), \quad z \in (0,l), \quad t \in (0,+\infty) \tag{2}$$

and boundary conditions

$$u(0,t) = d_1(t), \quad u(l,t) = d_2(t), \quad d_i \in C^1(\mathbb{R}) \cap L^\infty(\mathbb{R}). \tag{3}$$

Here $\mathcal{A}$ is linear Sturm-Liouville operator.

Problem 2. For $\alpha \in (0,\infty)$ we consider the following nonlinear differential equation

$$u_{tt}(z,t) + 2\alpha u_t(z,t) - u_{zz}(t,z) = f(u(z,t)) + D(z,t), \quad (z,t) \in (0,1) \times (0,+\infty) \tag{4}$$

with initial conditions

$$\begin{aligned} u(z,0) = \varphi_0(z), \quad u_t(z,0) = \varphi_1(z), \\ \varphi_0 \in H(0,1), \varphi_1 \in L^2(0,1), \end{aligned} \tag{5}$$

and with boundary conditions

$$u(0,t) = d_1(t), \quad u(1,t) = d_2(t), \quad d_i \in C^2(\mathbb{R}_+) \cap L^\infty(\mathbb{R}_+), \tag{6}$$

where $f \in C(\mathbb{R};\mathbb{R})$, $D \in C([0,\infty); L^2(0,1))$, $d_i(0) = 0$.

In this presentation using direct Lyapunov method [1] and new methods of construction Lyapunov functions for linear periodic systems [2] we got ISS properties and ISS estimations for solutions of this problems.

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References

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