Stability of linear switched impulsive systems with unstable subsystems

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We consider a Cauchy problem for a linear switched impulsive system [1]

$$\begin{aligned} \frac{dx}{dt} &= A_{\sigma(t)}x(t), \quad t \in (\tau_k, \tau_{k+1}), \\ \Delta x(t) &= B_{\sigma(t)}x(t), \quad t = \tau_k, \quad x(t_0) = x_0, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $\sigma(t)$ is a left continuous piecewise constant function which values belonging to the finite set $\{1, \dots, N\}$, $A_m \in \mathbb{R}^{n\times n}$, $m = 1, \dots, N$, $\{\tau_k\}_{k=1}^{\infty} \subset \mathbb{R}$ is an increasing sequence of moments of impulsive action having a single concentration point at infinity, $t_0 < \tau_1$, $x_0 \in \mathbb{R}^n$, $\Delta x(t) = x(t+0) - x(t)$, $B_m \in \mathbb{R}^{n\times n}$, $m = 1, \dots, N$.

Let us define the structural sets of the linear impulsive system $\mathcal{A} = \{A_1, \dots, A_N\}$, $\mathcal{B} = \{B_1, \dots, B_N\}$. With each pair of matrices $(A_m, B_m) \in \mathcal{A} \times \mathcal{B}$ we will associate a positive number θ_m (residence time), so if $\sigma(\tau_k) = m$, then $\tau_{k+1} - \tau_k = \theta_m$. The triple (A_m, B_m, θ_m) defines the subsystem of the hybrid system (1)

$$\begin{aligned} \frac{dz}{dt} &= A_m z(t), \quad t \neq k\theta_m, \\ \Delta z(t) &= B_m z(t), \quad t = k\theta_m, \quad z(t_0) = z_0. \end{aligned} \tag{2}$$

Note that the system (1) is not assumed to be periodic, so Floquet's theory is not applicable in this case.

We propose a new method for studying the stability of the hybrid system (1) for the case when all the matrices of the set $\mathcal{A}$ do not satisfy the Routh-Hurwitz condition, matrices from $\mathcal{B}$ do not satisfy the Schur's condition, and the subsystems (2) are all unstable. The proposed method of investigation is based on the ideas of commutator calculus [2].

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