Stability of Caputo non-instantaneous impulsive fractional differential equations with delays

SNEZHANA HRISTOVA[1], RAVI AGARVAL[2,3], RONAL O'REGAN[4]

[1]Plovdiv University, Plovdiv, Bulgaria
[2]Texas A&M University-Kingsville, Kingsville, TX 78363, USA
[3]Florida Institute of Technology, Melbourne, FL 32901, USA
[4]National University of Ireland, Galway, Ireland

emails: [1]snehri@gmail.com; [2]agarwal@tamuk.edu;
[3]donal.oregan@nuigalway.ie

Impulsive differential equations arise from real world problems to describe the dynamics of processes in which sudden, discontinuous jumps occur. Such processes are natural in biology, physics, engineering, etc. In the literature there are two popular types of impulses:

- *instantaneous impulses*- the duration of these changes is relatively short compared to the overall duration of the whole process;
- *non-instantaneous impulses* - an impulsive action, which starts abruptly at a fixed point and its action continues on a finite time interval.

In this talk Caputo fractional differential equations with non-instantaneous impulses and bounded delays are studied. Both basic approaches in the interpretation of the solutions of the fractional equation deeply connected with the presence of non-instantaneous impulses are discussed and illustrated on several examples.

There are several approaches in the literature to study stability, one of which is the Lyapunov approach. Some difficulties have been encountered when one applies the Lyapunov technique to Caputo fractional differential equations. The basic question which arises is the definition of the derivative of the Lyapunov like function along the given fractional equation. Initially a brief overview of the basic fractional derivatives of Lyapunov functions used in the literature is given and their advantages/disadvantages are discussed and illustrated on examples. Lyapunov functions and Razumikhin technique are applied to study stability properties of Caputo fractional differential equations with non-instantaneous impulses and bounded delays. Comparison results using this definition and scalar fractional differential equations are presented and several sufficient conditions for stability, uniform stability, asymptotic stability, Mittag-Leffler stability are established. Several examples are given to illustrate the theory. Also some applications to neural networks with bounded delays and impulsive perturbations acting as non-instantaneous impulses are presented.

MSC 2010: 34A08, 34K37, 34A37, 34K20
Keywords: Caputo fractional derivative, non-instantaneous impulses, Lyapunov functions, Caputo fractional Dini derivative, Razhumikhin method
Acknowledgement: Research was partially supported by the Fund NPD, Plovdiv University, No. FP17-FMI-008.