Parametrisation techniques for boundary value problems

ANDRÁS RONTÓ

Czech Academy of Sciences, Brno, Czech Republic

emails: ronto@math.cas.cz

The talk is based on joint works with I. Rachůnková, L. Rachůnek, M. Rontó, N. Shchobak, and J. Varha.

We discuss parametrisation techiques for the investigation of boundary value problems for systems of ordinary differential equations

$$u'(t) = f(t, u(t)), \qquad t \in [a, b],$$

with $-\infty < a < b < \infty$ and f Lipschitzian on a bounded set. The approach uses Lyapunov-Schmidt type reductions combined with successive approximations and allows one to efficiently construct approximate solutions and, in many cases, establish the solvability of the problem in a rigorous way using the results of computation. Suitable versions of the technique allow one to deal with the periodic boundary conditions, more general two-point conditions (possibly, non-linear)

$$g(u(a), u(b)) = 0$$

and non-local conditions

$$\phi(u) = \gamma.$$

The applicability of the approach is ensured by the assumption that f has sufficently small Lipschitz constant on a sufficiently wide set. We discuss an interval halving technique allowing one to overcome this limitation in a certain sense.

The techniques are illustrated by numerical examples. We also mention applications to other problems and combination with other approaches.

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