PROJECTIVE GEOMETRY RELATED TO THE SECANT LOCI IN SYMMETRIC PRODUCT OF SMOOTH ALGEBRAIC CURVES

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Abstract

We describe the tangent space of the secant loci associated to a line bundle on a projective smooth algebraic curve. Denoting by $V_d^r(L)$ the $(d-r)$-th secant loci of C associated to the line bundle L on C, we obtain:

Theorem 0.1 *(a) If D belongs to $V_d^r(L) \setminus V_d^{r+1}(L)$, the tangent space to $V_d^r(L)$ at D is*

$$T_D(V_d^r(L)) = (Im(\alpha_L \mu_0^L))^\perp$$

where μ_0^L is the cup product map

$$\mu_0^L : H^0(C, \mathcal{O}(D)) \otimes H^0(C, L(-D)) \to H^0(C, L).$$

(b) If $D \in V_d^{r+1}(L)$ then $T_D(V_d^r(L)) = H^0(C, L \otimes \mathcal{O}_D)$. In particular, if $V_d^r(L)$ has the expected dimension and $d < s + 1 + r$, then $D \in \mathrm{Sing}(V_d^r(L))$.

Theorem 0.2 *The scheme $V_d^r(L)$ is smooth at $D \in V_d^r(L) \setminus V_d^{r+1}(L)$ and has the expected dimension $d - r \cdot (s+1-(d-r))$ if and only if μ_0^L is injective.*

Lemma 0.3 *For a very ample line bundle L on C and an integer d with $d \geq 4$, if $V_d^r(L) \neq \emptyset$, then no irreducible component of $V_d^r(L)$ is contained in $V_d^{r+1}(L)$.*

Theorem 0.4 *Let C be a hyper-elliptic curve and L a line bundle on C whose space of global sections has dimension $s + 1$. Assume moreover that $d \leq s + 1$. Then $V_d^r(L)$ is empty or irreducible of dimension $d - r$ according to whether $d < 2r$ or $2r \leq d$, respectively.*

Theorem 0.5 *If C is non hyper-elliptic and L a very ample line bundle on C with $d \leq h^0(L) - 1$, then every component of $V_d^r(L)$ has dimension at most equal to $d - r - 1$.*

Corollary 0.6 *Assume that L is a very ample line bundle on C with $h^0(L) = d + 1 \geq 4$. Then $V_d^1(L)$, if non empty, is of dimension $d - 2$.*

Keywords: Symmetric Products; Very Ample Line Bundle.

References

[1] M. Aprodu, E. Sernesi; Secant spaces and syzygies of special line bundles on curves, Algebra and Number Theory **9**, (2015), 585–600.

[2] A. Bajravani; Martens-Mumford Theorems for Brill-Noether Schemes arising from Very Ample Line bundles, To appear in Archive der Mathematik.

[3] M. Coppens; Some remarks on the Scheme W_d^r, Annali di Matem. P. e A. **97**(1990), 183–197.

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