# HYPERFUNCTION METHOD FOR NUMERICAL INTEGRATIONS

**Hidenori Ogata**[1], **Hiroshi Hirayama**[2]

1 *The University of Electro-Communications, Chofu, Japan*
2 *Kanagawa Institute of Technology, Atsugi, Japan*

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## Abstract

In this paper, we examine a numerical integration method proposed by Hirayama [1]. In his method, an integral $I = \int_a^b f(x)w(x)\mathrm{d}x$ ( $-\infty < a < b < +\infty$ ), where $f(x)$ is a given real analytic function and $w(x)$ is a weight function, is transformed into the complex integral on a closed contour

$$I = \frac{1}{2\pi \mathrm{i}} \oint_C f(z)\Psi(z)\mathrm{d}z \quad \text{with} \quad \Psi(z) = \int_a^b \frac{w(x)}{z-x}\mathrm{d}x, \tag{1}$$

where $C$ is a closed contour surrounding the inverval $[a, b]$ and included in a complex domain $D$ such that $f(z)$ is analytic in it, and is approximated by the trapezoidal rule. We here call this method the "hyperfunction method" since (1) is the definition of the integral $I$ when the integrand $f(x)w(x)$ is regarded as a hyperfunction [2]. The hyperfunction method gives good approximations especially for integrals with so strong end-point singularities that the DE rule [3] does not work for them.

**Keywords:** Numerical integration, analytic function, hyperfunction

## References

[1] H. Hirayama, A numerical integration rule by the contour integral transformation, Abstracts of the 44-th Numerical Analysis Symposium, 21–24 (in Japanese).

[2] U. Graf, Introduction to Hyperfunctions and Their Integral Transforms — An Applied and Computational Approach —, Birkhäuser, Basel, 2010.

[3] H. Takahasi and M. Mori, Double exponential formulas for numerical integrations, Publ. RIMS Kyoto Univ. **9** (1974) 721–741.

[1]e-mail: ogata@im.uec.ac.jp
[2]e-mail :hirayama@sd.kanagawa-it.ac.jp