THE SKEW INVERSE SEMIGROUP RING

Sahar Moayeri Rahni[1]

[1] *Shiraz University, Shiraz, Iran*

MSC 2000: 20M18, 16W22

Abstract

For a given partial action π of an inverse semigroup S on an associative algebra $\mathcal{A}$, we introduce the notation of *skew inverse semigroup ring* $\mathcal{A} \rtimes_\pi S$, and prove that this construction is associative algebra under some conditions on a partial action π. At the end we define the concept of *strongly associative algebra* and we show that a semiprime algebra $\mathcal{A}$ is strongly associative. We refer to the treatises [1, 2, 3] for a thorough treatment of the concepts of partial actions, actions, and crossed products. Let $\pi = (\{\pi_s\}_{s\in S}, \{X_s\}_{s\in S})$ be a partial action of S on $\mathcal{A}$, and let $L = \{\sum_{s\in S} a_s\delta_s \ : \ a_s \in X_s\}$ the set of all formal finite sums, with the following multiplication:

$$(a_s\delta_s).(b_t\delta_t) = \pi_s(\pi_{s^*}(a_s)b_t)\delta_{st}.$$

With the aid of multiplier algebra, instead of using approximate identity of C^*-algebra as in [3], we will prove that if for each $s \in S$ the ideal X_s is (L, R)-associative then L is associative, so, it is an algebra. Let I be the ideal generated by the set $\{a\delta_r - a\delta_t \ : \ \text{where } r \leq t \text{ and } a \in X_r\}$, then $\mathcal{A} \rtimes_\pi S$ is the quotient algebra $\frac{L}{I}$, hence, it is an associative algebra.

Keywords: Partial action, Inverse semigroup, Multiplier Algebra3

References

[1] Buss, A. , Exel, R.: Inverse Semigroup Expansions and Their Actions on C^*-Algebras, *Illinois. Math.*, Volume 56, Number 4, Winter 2012, Pages 1185-1212.

[2] Exel, R: Inverse semigroups and Combinatorial C^*-Algebra, *Bull. Braz. Math. Soc. (N.S.)*, **39**, (2008) ,191–313.

[3] N. Sieben, *C^*-Crossed Products by Partial Actions and Actions of Inverse semigroups* , J. Austral. Math. Soc. (Series A) **63** (1997), 32–46.

[1]e-mail: smoayeri@shirazu.ac.ir