THE (C, α, β) INTEGRABILITY OF FUNCTIONS AND A TAUBERIAN THEOREM

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Abstract

For a continuous function $f(T, S)$ on $\mathbb{R}_+^2 = [0, \infty) \times [0, \infty)$, we define its integral on $\mathbb{R}_+^2$ by

$$F(T, S) = \int_0^T \int_0^S f(t, s)dtds,$$

and its (C, α, β) mean by

$$\sigma_{\alpha,\beta}(T, S) = \int_0^T \int_0^S \left(1 - \frac{t}{T}\right)^\alpha \left(1 - \frac{s}{S}\right)^\beta f(t, s)dtds,$$

where $\alpha > -1$, and $\beta > -1$. We say that $\int_0^\infty \int_0^\infty f(t, s)dtds$ is (C, α, β) integrable to L if $\lim_{T,S\to\infty} \sigma_{\alpha,\beta}(T, S) = L$ exists.

We prove that if $\lim_{T,S\to\infty} \sigma_{\alpha,\beta}(T, S) = L$ exists for some $\alpha > -1$ and $\beta > -1$, then $\lim_{T,S\to\infty} \sigma_{\alpha+h,\beta+k}(T, S) = L$ exists for all $h > 0$ and $k > 0$.

Next, we prove that if $\int_0^\infty \int_0^\infty f(t, s)dtds$ is $(C, 1, 1)$ integrable to L and

$$T \int_0^S f(T, s)ds = O(1)$$

and

$$S \int_0^T f(t, S)ds = O(1)$$

then $\lim_{T,S\to\infty} F(T, S) = L$ exists.

Keywords: The (C, α, β) integrability, improper double integral, convergence in Pringsheim's sense, Tauberian conditions and theorems.

References

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