SOME ERGODIC PROPERTIES OF MEASURES

Heybetkulu Mustafayev[1]

Department of Mathematics, Faculty of Science, Yüzüncü Yıl University, 65080, Van, TURKEY

MSC 2000: 16S34, 16U60

Abstract

Let G be a locally compact abelian group with the dual group Γ and let $M(G)$ be the convolution measure algebra of G. By $\widehat{\mu}$ we denote the Fourier-Stieltjes transform of $\mu \in M(G)$:

$$\widehat{\mu}(\gamma) = \int_G \overline{\gamma}(g)\, d\mu(g),\ \gamma \in \Gamma.$$

For $n \in \mathbb{N}$, by μ^n we denote $n-$times convolution power of $\mu \in M(G)$. A measure $\mu \in M(G)$ which satisfies $\sup_{n\in\mathbb{N}} \|\mu^n\| < \infty$ is called *power bounded.*

In the case when $1 < p \leq 2$, by $\widehat{f}$ we will denote the Hausdorff-Young-Plancherel transform of $f \in L^p(G)$. For a closed subset F of Γ, by $L^p(F)$ we denote the set of all $f \in L^p(G)$ such that $\widehat{f} = 0$ almost everywhere on F ($\widehat{f}$ is only defined up to sets of Haar measure zero).

We have the following.

Theorem. Let G be a locally compact abelian group and let μ ba a power bounded measure on G. If $1 < p \leq 2$, then the following conditions are equivalent for a closed subset F of Γ:

(a) $\lim_{n\to\infty} \left\| \frac{1}{n} \sum_{k=0}^{n-1} \mu^k * f \right\|_p = 0$, for all $f \in L^p(F)$.

(b) $\widehat{\mu}(\gamma) \neq 1$, for all $\gamma \in \Gamma \setminus F$.

References

[1] U. Krengel, Ergodic Theorems, Walter de Gruyter, Berlin New York, 1985.

[2] B. M. Schreiber, Measures with bounded convolution powers, Trans. Amer. Math. Soc. **151** (1970) 405-431.

[3] R. Jones, J. Rosenblatt, A. Tempelman, Ergodic theorems for convolutions of a measure on a group, Illinois J. Math., **38** (1994) 521-553.

[4] E. Kaniuth, A.T. Lau, A. Ülger, Power boundedness in Fourier and Fourier-Stieltjes algebras and other commutative Banach algebras, J. Funct. Anal. **260** (2011) 2366-2386.

[1]hsmustafayev@yahoo.com