RECENT TRENDS RELATED TO FOUR DIMENSIONAL MATRIX TRANSFORMATIONS

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Abstract

Our main goal is to present a short survey on the spaces of double sequences and four dimensional matrix transformations. In Section 2, we give the corresponding results for four dimensional dual summability methods of the new sort to the results obtained by Altay and Başar in [*Some paranormed Riesz sequence spaces of non-absolute type*, Southeast Asian Bull. Math. **30** (5) (2006), 591–608] for two dimensional dual summability methods of the new sort. In Section 3, we present multidimensional analogues of Petersen' theorem "The necessary and sufficient conditions for the regular matrix $A = (a_{mn})$ to be stronger than the regular Riesz mean (R, p_n), where $p_n > 0$ for all $n \in \mathbb{N}$" for double sequences. In Section 4, as the domain of four dimensional Riesz mean R^{qt} associated with the sequences $q = (q_k)$ and $t = (t_l)$ of non-negative real numbers in the spaces $\mathcal{M}_u$, $\mathcal{C}_p$, $\mathcal{C}_{bp}$ and $\mathcal{C}_r$, we introduce the double sequence spaces $R^{qt}(\mathcal{M}_u)$, $R^{qt}(\mathcal{C}_p)$, $R^{qt}(\mathcal{C}_{bp})$ and $R^{qt}(\mathcal{C}_r)$, and also examine some properties of those sequence spaces. Furthermore, we show that these sequence spaces are Banach spaces. Let $\vartheta \in \{p, bp, r\}$. We determine the α-dual and $\beta(\vartheta)-$dual of the space $R^{qt}(\mathcal{M}_u)$ and $\beta(\vartheta)-$duals of the spaces $R^{qt}(\mathcal{C}_{bp})$ and $R^{qt}(\mathcal{C}_r)$ of double sequences. Finally, we characterize the classes $(R^{qt}(\mathcal{C}_r) : \mathcal{C}_\vartheta)$, $(\mu : (R^{qt}(\mathcal{C}_\vartheta))$ and $(R^{qt}(\mathcal{C}_\vartheta) : \mathcal{C}_f)$ of four-dimensional matrix transformations, where μ and $\mathcal{C}_f$ denote any given double sequence space and the space of almost convergent double sequences, respectively. Section 5 is devoted to Steinhaus type theorems together with the definitions of four dimensional conull and coregular matrices, and the characterizations of the classes $(\mathcal{M}_u : \mathcal{C}_\vartheta)$, where $\vartheta \in \{p, p0, f\}$. In Section 6, we state and prove the Mercerian theorem for a four dimensional matrix and the space of convergent double sequences in the Pringsheim's sense.

Keywords: Double sequence space, paranormed sequence space, alpha-, beta-duals and matrix transformations.

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