

TAUBERIAN CONDITIONS FOR THE (C, α) INTEGRABILITY OF
FUNCTIONS

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Abstract

For a real-valued continuous function $f(x)$ on $[0, \infty)$, we define

$$s(x) = \int_0^x f(u)du \text{ and } \sigma_\alpha(x) = \int_0^x \left(1 - \frac{u}{x}\right)^\alpha f(u)du$$

for $x > 0$. We say that $\int_0^\infty f(u)du$ is (C, α) integrable to L for some $\alpha > -1$ if the limit $\lim_{x \rightarrow \infty} \sigma_\alpha(x) = L$ exists.

It is known that $\lim_{x \rightarrow \infty} s(x) = L$ implies $\lim_{x \rightarrow \infty} \sigma_\alpha(x) = L$ for all $\alpha > -1$. The aim of this paper is twofold. First, we introduce some new Tauberian conditions for (C, α) integrability method under which the converse implication is satisfied and improve classical Tauberian theorems for the (C, α) integrability method. Next, we give short proofs of some classical Tauberian theorems as special cases of some of our results.

Keywords: Divergent integrals, Cesàro integrability, (C, α) integrability, Tauberian theorems

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