STABILITY AND BOUNDEDNESS OF SOLUTIONS OF VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract

In 2003, Vanualailai and Nakagiri [2] considered the nonlinear Volterra integro-differential equation without delay

$$\frac{d}{dt}[x(t)] = A(t)f(x(t)) + \int_{0}^{t} B(t,s)g(x(s))ds, \tag{1}$$

where $t \geq 0$, $x \in \Re$, $A(t) : [0,\infty) \to (-\infty, 0)$, $f,\ g : \Re \to \Re$ are continuous functions, and $B(t,s)$ is a continuous function for $0 \leq s \leq t < \infty$. Vanualailai and Nakagiri [2] studied the stability of solutions of equation (1) by defining a suitable Lyapunov functional.

In this paper, we consider the nonlinear Volterra integro-differential equation with delay

$$x'(t) = -a(t)f(x(t)) + \int_{t-\tau}^{t} B(t,s)g(x(s))ds + p(t), \tag{2}$$

where $t \geq 0$, τ is a positive constant, fixed delay, $x \in \Re$, $a(t) : [0,\infty) \to (0,\infty)$, $p : [0,\infty) \to \Re, f,\ g : \Re \to \Re$ are continuous functions with $f(0) = g(0) = 0$, $B(t,s)$ is a continuous function for $0 \leq s \leq t < \infty$. We investigate the stability of zero solution and boundedness of solutions of equation (2) by defining suitable Lyapunov functionals, when $p(t) \equiv 0$and $p(t) \neq 0$, respectively.

Keywords: Stability and boundedness, Volterra integro-differential equations, Lyapunov functionals

References

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