

A NUMERICAL METHOD FOR NONLINEAR SINGULARLY
PERTURBED MULTI-POINT BOUNDARY VALUE PROBLEM

Musa Cakir¹, Derya Arslan²

^{1,2}Yuzuncu Yil University, Van, Turkey

MSC 2000: 34B10, 65L05, 65L11, 65L12, 65L20

Abstract

We consider the following nonlinear singular perturbed multi-point problem:

$$\begin{aligned} -\varepsilon^2 u'' + f(x, u) &= 0, \quad 0 < x < 1, \\ u(0) &= 0, \\ k_0 u(1) &= \sum_{i=1}^m k_i u(s_i) + k_{m+1} \int_0^1 u(x) dx + d, \end{aligned}$$

where $0 < \varepsilon \ll 1$ is small perturbation parameter, the function $f(x, u)$ is sufficiently smooth on $[0, 1] \times \mathbb{R}$, $s_i \in (0, 1)$, $i = 1, 2, \dots, m$, $k_0 \geq 0$, and furthermore $\frac{\partial f}{\partial u}(x, u) \geq \alpha > 0$. The solution $u(x)$ has boundary layers at $x = 0$ and $x = 1$. This study is concerned with ε -uniform numerical method for the nonlinear singularly perturbed multi-point boundary value problem. We describe some properties of the solution of this problem. The numerical method is constructed on Shishkin mesh and the method is shown to be convergent of first order in the discrete maximum norm. Consequently, the numerical experiments which demonstrate the sharpness of our theoretical analysis are presented.

Keywords: Singular perturbation; Fitted finite difference method ; Shishkin mesh; Nonlocal boundary condition; Uniform convergence

References

- [1] A. A. Smarskii, Theory of Difference Scheme. Marcel Dekker, New York, (2001).
- [2] R. Cziegis, The Numerical Solution of Singularly Perturbed Nonlocal Problem, Lietuvas Matem. Rink. (in Russian), 28(1988)144-152.
- [3] D. Herceg, On The Numerical Solution of a Singularly Perturbed Nonlocal Problem, Univ. u Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat., 20(1990)1-10.

¹First Author's e-mail: cakirmusa@hotmail.com

²Second Author's e-mail: ayredlanu@gmail.com