SHARP RUSAK-TYPE INEQUALITIES FOR RATIONAL FUNCTIONS ON SEVERAL INTERVALS

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Abstract

We consider sharp Rusak-type inequalities for rational functions on several intervals when the system of intervals is a "rational function inverse image" of an interval and those functions are large in gaps.

Let $\Re(\xi_1, \ldots, \xi_{2n})$ be the set of all " rational functions "of the form $r(x) = \frac{b_0 x^n + b_1 x^{n-1} + \ldots + b_n}{\sqrt{\rho_\nu(x)}}$, $b_0, \ldots, b_n \in \mathbb{C}$ and $\rho_\nu(x) = \prod_{j=1}^{2n}(x - \xi_j)$ is a real polynomial of degree ν which is positive on $E = \bigcup_{j=1}^{l}[a_{2j-1}, a_{2j}]$, $-1 = a_1 < a_2 < \ldots < a_{2l} = 1$. ($\xi_j$ might be equal to ∞, then $(x - \xi_j)$ should be omitted) Consider also the set $\Re^*(\xi_1, \ldots, \xi_{2n})$ which consists of those functions $r \in \Re(\xi_1, \ldots, \xi_{2n})$, which satisfy $|r(x)| > \|r\|_{C(E)}$ for all $x \in [-1, 1] \backslash E$. The last condition can not omit.

Theorem. Suppose that $\sum_{j=1}^{2n} \omega_k(\xi_j) = 2q_k$, $q_k \in \mathbb{N}$, $k = 1, \ldots, l$, and $|\xi_j| > 1, j = 1, \ldots, 2n$. Then for any $r \in \Re^*(\xi_1, \ldots, \xi_{2n}), \|r\|_{C(E)} = 1$ the inequality

$$|r'(x)| \leq \begin{cases} \gamma_n'(x), x \in \widetilde{E}_n, \\ |m_n'(x)|, x \in E \backslash \widetilde{E}_n \end{cases} \tag{1}$$

is valid, where

$$\begin{aligned} m_n(x) &= \cos(\gamma_n(x)), \gamma_n(x) = \frac{\pi}{2} \int_{a_1}^{x} \sum_{j=1}^{2n} \varpi_E(x, \xi_j)\, dx, \\ \widetilde{E}_n &= [x_1, x_{q_1}] \cup [x_{q_1}, x_{q_1+q_2}] \cup \ldots \cup [x_{q_1+\ldots+q_{l-1}}, x_n], \end{aligned}$$

and $x_1 < \ldots < x_n$ are zeros of m_n (there are q_k zeros on $[a_{2k-1}, a_{2k}], k = 1, \ldots, l$).

For $r(x) \equiv \varepsilon m_n(x)$, $|\varepsilon| = 1$, inequality in (1) is attained.

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